EXPECTED STOCK RETURNS

Ana González-Urteaga

Department of Business Management, Universidad Pública de Navarra, Campus Arrosadía, 31006 Pamplona, Spain. Phone: (34) 948169400 (ana.gonzalezu@unavarra.es)

Belén Nieto

Department of Financial Economics and Accounting, Universidad de Alicante, San Vicente del Raspeig, 03690, Alicante, Spain. Phone: (34) 965903400 (belen.nieto@ua.es)

Gonzalo Rubio

Department of Economics and Business, Universidad CEU Cardenal Herrera, Reyes Católicos 19, 03204 Elche, Alicante, Spain. Phone: (34) 965426486 (gonzalo.rubio@uch.ceu.es)

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Abstract

Contrary to the standard practice of using past average realized returns when testing asset pricing models, this paper analyzes the factor structure and the cross-sectional variability of a set of expected returns, which are shown to have well-behaved time-varying characteristics. We show that the first two principal components explain 99.6% of the variability of (lower bound) expected returns. Quality, funding illiquidity, the default premium and the market-wide variance risk premium explain most of the time-varying behavior of the first principal component. A multi-factor model with the market, and the four aggregate factors that explain the first principal component significantly explains the cross-sectional variability of (lower bound) expected excess returns. The cross-sectional *R*-squares proposed by Kan, Robotti, and Shanken (2013) for the principal component and the multifactor models are 52% and 84%, respectively. Both measures of fit are (asymptotically) different from zero.

Keywords: expected returns; risk-neutral variance; factor structure of expected returns; cross-section of expected returns

JEL classification: G12, G13

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Corresponding author: Gonzalo Rubio (gonzalo.rubio@uch.ceu.es)

1. Introduction

Alternative empirical specifications of macroeconomic-based asset pricing models have shown that expected returns are time-varying and show a clear counter-cyclical behavior.[1] As reported by Harvey, Liu, and Zhu (2016), the traditional cross-sectional analysis of factor-based models has delivered 316 systematic factor risks capable of explaining average returns. It is somehow frustrating that after years of academic research the important and obvious empirical question remains open: How many state variables do we need to price financial assets? It is important to emphasize that this huge literature employs average realized returns as proxies for expected returns.

The key insight is raised in Cochrane (2017), who points out that the truly relevant question is simply: What is the factor structure of expected returns? This is very different from the well-known research regarding the factor structure of realized returns at the end of the period. Along this line, our paper investigates the factor structure of returns that are expected at the beginning of the period. This issue is surprisingly unexplored in the empirical literature of financial economics, and it is the main research objective of this paper. In addition, we analyze the time-series and cross-sectional variability of expected returns by taking advantage of the evidence obtained from the factor analysis.

The old question of how to estimate expected returns remains open. It is not surprising that researchers have then employed average realized returns over long horizons to proxy the true expected returns. However, the seminal paper of Breeden and Litzenberger (1978) shows that we may use option prices to extract forward-looking risk-neutral probabilities. As Kadan and Tang (2016) point out, it remains much more controversial whether we can extract physical probabilities from option prices and be able

[1] See Cochrane (2007, 2011, and 2017) for detailed reviews.

to derive the implied expected returns. Indeed, the finance profession has recently witnessed the debate raised by the Recovery theorem of Ross (2015), and the follow up papers by Borovicka, Hansen, and Scheinkman (2016), Bakshi, Chabi-Yo, and Gao (2016), Jackwerth and Menner (2016), and Jensen, Lando, and Pedersen (2017).

Using an alternative and less ambitious approach, Martin (2013 and 2017) obtains a lower bound for the expected market risk premium by extracting forward-looking information from option data and, in particular, from risk-neutral variances. The cost of this approach is that Martin (2017) does not obtain full recovery but, at least, he is able to obtain a useful lower bound on the expected market excess return. Our paper employs this approach to study the factor structure of the lower bound of expected excess returns at the individual level, and the determinants of their temporal and cross-sectional variability.

However, even before the start of the principal component analysis and the factor structure of expected returns, it is important to understand the relation between the (lower bound) expected returns and future realized returns. Our evidence shows that our proxies for expected returns are reasonably related to future realized returns at the one-month horizon. Note that one month is the maturity of the options from which we extract expected returns. Therefore, the one-month horizon is precisely the appropriate horizon to relate expected returns and future realized returns.

We use the traditional principal component analysis and the methodology of Connor and Korajczyk (1988) to extract the factors that better explain the variability of the variance-covariance matrix of (lower bound) expected excess returns of 20 risk-neutral variance-sorted portfolios. Independently of the technique employed, we show that the first two principal components are enough to capture approximately 99.6% of the variability of expected excess returns.[2] In contrast, when using realized returns, the first two principal components only explain around 77% of their variability. A particularly relevant question is to uncover the underlying determinants of the two first principal components of the factor structure of expected excess returns. We employ well-known aggregate risk factors, which have been shown to be relevant priced factors in fully recognized previous research.[3] The first principal component is strongly explained by the default premium, the quality minus junk factor (QMJ) of Asness, Frazzini, and Pedersen (2014), the market variance risk premium, and the betting against beta factor (BAB) of Frazzini and Pedersen (2014). These variables explain 53.4% of the temporal variability of the first principal component. On the other hand, the momentum factor of Carhart (1997) is the only significant factor that explains the temporal behavior of the second component with a positive sign. Moreover, the highest adjusted *R*-square across alternative combinations of factors is low and equal to 6.1%.

In addition, we analyze the cross-sectional variability of (lower bound) expected excess returns. We first employ the two first principal components to find that their betas, using the traditional cross-sectional *R*-square statistic, explain 90.9% of the cross-sectional variability. We also use a multi-factor asset pricing model, which includes the excess market return and the four factors that explain the time-varying behavior of the first principal component of expected returns. The betas of these five factors are significantly priced with the correct theoretical sign, and they jointly explain 98.2% of the cross-sectional variability of (lower bound) expected excess returns. Moreover, we employ the standard errors suggested by Kan, Robotti, and Shanken (2013), which are adjusted by errors-in-variable and model misspecification, and the corresponding

[2] Indeed, the first component by itself explains around 97% of their variability.
[3] It is important to point out that we do not fish for factors. We guide our initial selection of candidates by using the eleven most popular and successful factors than have been employed in the analysis of the cross-sectional variability of past average returns.

corrected *R*-square. The cross-sectional results remain valid, with *R*-squares of 52.3% and 83.6% for the principal components and the multi-factor model, respectively. These modified *R*-squares are in both cases (asymptotically) statistically different from zero.

Several robustness analyses are carried out. We use an alternative sorting procedure to extract (lower bound) expected excess returns. Portfolios sorted by market betas tend to produce similar results, although the default premium losses significance. We also evaluate exact expected returns rather than lower bounds. Under the exact relation, expected returns incorporate the covariance between returns and the product of returns and the stochastic discount factor. To avoid a model dependent stochastic discount factor, we employ the non-parametric estimation procedure of the Information Stochastic Discount Factor suggested by Ghosh, Julliard, and Taylor (2016 a, b). Our analysis suggests that the lower bound approximation works reasonable well. Overall, we find parallel but somewhat weaker results than the ones reported under the lower bound approximation.[4] Finally, we approximate risk-neutral variances following the alternative approach of Britten-Jones and Neuberger (2000), and Jiang and Tiang (2005). These authors use a different weighting scheme to extract information from option data. Overall, we find very similar results independently of the estimation methodology employed in the analysis.

This paper proceeds as follows. Section 2 describes the theoretical framework to extract information about expected returns from option data, and Section 3 discusses the data employed in the research. Section 4 presents the estimation and descriptive statistics of (lower bound) expected excess returns. Section 5 presents the relation between our set of ex-ante returns and the corresponding ex-post realized returns. In Section 6, we discuss

[4] The drawback of this approach is that the estimated Information Stochastic Discount Factor depends on the returns of the assets employed in its estimation. Therefore, the cross-sectional results may be sensitive to the particular sample of assets involved in the estimation.

the factor structure of (lower bound) expected excess returns, and Section 7 contains the analysis of their cross-sectional variability. Section 8 displays the empirical results under an alternative sorting procedure. In Section 9, we estimate exact (rather than lower bound) expected excess returns. Finally, Section 10 presents our conclusions. At the end of the paper, the Appendix contains the results under an alternative estimation procedure of expected returns.

2. The Lower Bound on the Expected Risk Premium

Martin (2017) obtains a lower bound on the expected market risk premium from the fundamental pricing equation for the market portfolio return:

$$1 = E_t^P\left(M_{t+1}R_{mt+1}\right) = \frac{E_t^Q\left(R_{mt+1}\right)}{R_{ft}} \quad , \tag{1}$$

where R_{mt+1} is the gross market return at time *t+1*, M_{t+1} is the stochastic discount factor (SDF) at time *t + 1*, R_{ft} is the gross risk-free rate available at time *t*, $E_t^P(\cdot)$ is the expectation operator under the physical probability as of time *t*, and $E_t^Q(\cdot)$ is the risk-neutral expectation operator as of time *t*.

The risk-neutral variance is

$$Var_t^Q\left(R_{mt+1}\right) = E_t^Q\left(R_{mt+1}^2\right) - \left(E_t^Q\left(R_{mt+1}\right)\right)^2 . \tag{2}$$

Combining expressions (1) and (2), after a simple manipulation, the expected market risk premium is

$$E_t^P\left(R_{mt+1}\right) - R_{ft} = \frac{1}{R_{ft}} Var_t^Q\left(R_{mt+1}\right) - Cov_t^P\left(M_{t+1}R_{mt+1}, R_{mt+1}\right). \tag{3}$$

Martin (2017) points out that, as long as the relative risk aversion coefficient is greater or equal than one, under mild conditions and for most theoretical models, the following negative correlation condition (NCC) holds for the market portfolio return

$$Cov_t^P\left(M_{t+1}R_{mt+1}, R_{mt+1}\right) \leq 0 . \tag{4}$$

Thus, the risk-neutral variance normalized by the risk free rate constitutes a lower bound for the expected market risk premium:

$$E_t^P\left(R_{mt+1}\right) - R_{ft} \geq \frac{1}{R_{ft}} Var_t^Q\left(R_{mt+1}\right). \tag{5}$$

Therefore, the advantage of the lower bound is that avoids estimating the covariance between returns and the product of returns and the stochastic discount factor. So, it avoids using a model-dependent SDF.

If the NCC is satisfied for individual stocks, the lower bound on expected returns for any given asset *j* is

$$E_t^P\left(R_{jt+1}\right) - R_{ft} \geq \frac{1}{R_{ft}} Var_t^Q\left(R_{jt+1}\right). \tag{6}$$

However, Kadan and Tang (2016) point out that the argument used by Martin (2017) does not extend to the case of individual stocks. For any given asset *j*, equation (4) can be written as[5]

$$Cov_t^P\left(U'\left(\sum_{i=1}^{N} \omega_i R_{it+1}\right) R_{jt+1}, R_{jt+1}\right) \leq 0 \ , \tag{7}$$

[5] Kadan and Tang (2016) extend this simple analysis to the cases of a dynamic model with separable utility, and a dynamic consumption model with recursive utility.

where $U'(\cdot)$ is marginal utility, and ω_i is the weight given to asset *i* in a portfolio composed of *N* assets. Therefore, the sign of the covariance in (7) depends on the entire correlation structure between R_{jt+1} and all other stocks. Kadan and Tang (2016) obtain a sufficient condition under which the NCC holds for individual stocks. Let γ be the relative risk aversion. Then, the sufficient condition for equation (6) to hold is

$$\gamma \geq \frac{Var_t\left(R_{jt+1}\right)}{Cov_t\left(R_{jt+1}, R_{mt+1}\right)} \equiv \delta_{jt}. \tag{8}$$

Thus, the NCC holds for any individual stock *j* with a positive beta if risk aversion is high enough to be larger than the stock's delta. The obvious problem with this condition is that risk aversion is not observable. However, we have ample evidence about the typical values of risk aversion, which for most individuals is between 1 and 10. Depending upon the values of δ_j, we can decide how binding the NCC is for a given particular application.

As a powerful alternative approach, Martin and Wagner (2018) also study expected excess returns at the individual level. They theoretically show the expected excess return on a stock is the risk-neutral variance of the market plus the stock's excess risk-neutral variance relative to the average stock. Although the empirical sections of the paper are about testing their theoretical model using forecasting-based regressions, it is also true that the fundamental insight comes from the extracting expected returns from option trading as shown by Martin (2017). In fact, the theoretical model of Martin and Wagner (2018) employs expression (5), although it also assumes that $Cov_t^P\left(M_{t+1}R_{mt+1}, R_{mt+1}\right) = 0$. Using a careful panel data approach with individual stocks and options written on these stocks with alternative maturities, they find a substantial support for their model, particularly over 6-, 12-, and 24-month horizons.[6]

3. Data

In order to obtain the lower bounds of expected returns, we have to extract the risk-neutral variance for any given asset j. As we explain in the next Section, we calculate risk-neutral variances by integrating option prices for alternative strike prices. We employ daily data from OptionMetrics for the S&P 100 Index options and for individual options on all stocks included in the S&P 100 Index at some point during the sample period from January 1996 to August 2015. This yields 201 stocks used in our estimations. From the OptionMetrics database, we obtain all put and call options on the individual stocks and on the index with time to maturity τ between six days and 60 days. Given that the options are American style, it is convenient to work with short-term maturity options, for which the early exercise premium tends to be negligible.[7] We select the best bid and ask closing quotes to calculate the mid-quotes as the average of bid and ask prices, not actual transaction prices, to avoid the well-known bid-ask bounce problem described by Bakshi, Cao, and Chen (1997). In selecting our final option sample, we apply the usual filters. We discard options with zero open interest, zero bid prices, missing option delta or implied volatility, and negative implied volatility. Regarding the exercise level, we follow Jiang and Tian (2005), Driessen, Maenhout, and Vilkov (2009), and Martin (2017) and exclude in-the-money options. In addition, we ignore options with extreme moneyness, that is, puts with a delta higher than -0.05 and calls with a delta lower than 0.05.

[6] Their approach also depends on whether data employed satisfy the assumptions involved in their expressions.
[7] See the evidence reported by Driessen, Maenhout, and Vilkov (2009), who employ a similar database.

Fama and French (2015) (FF hereafter) show that a five-factor model, that expands their popular three-factor model with profitability (robust minus weak, RMW) and investment (aggressive minus conservative, CMA) factors, explains anomalies associated with low betas, low share repurchases, and low volatility assets relative to high betas, high repurchases, and high volatility securities.[8] On the other hand, they are not able to explain the cross-section variability of momentum portfolios unless the momentum factor (MOM) of Carhart (1997) is included in the cross-section. Thus, from Kenneth French's website (http://mba.tuck.darmouth.edu), we collect monthly data on the FF five factors, the value-weighted stock market portfolio return, the risk-free rate, the MOM factor, the 25 FF portfolios by size and book-to-market, the 32 FF portfolios sorted by size, book-to-market and profitability, and the 10 portfolios sorted by investment aggressiveness. We also collect daily and monthly data on the 25 FF portfolios by size and investment aggressiveness, the 25 FF portfolios by book-to-market and profitability and the 10 portfolios sorted by momentum.

We also use the QMJ factor of Asnes et al. (2014), further explored recently by Asness, Frazzini, Israel, Moskowitz, and Pedersen (2017). These authors define a quality stock as an asset for which an investor would be willing to pay a higher price. These are stocks that are safe (low required rate of return), profitable (high return on equity), growing (high cash flow growth), and well managed (high dividend payout ratio). Asnes et al. (2014) show that the QMJ factor, that buys high-quality stocks and shorts low-quality (junk) stocks, earns significant risk-adjusted returns not only in the U.S. market, but also in 24 other countries. The QMJ factor is downloaded from AQR Capital Management Database (www.aqr.com).

[8] Novy-Marx (2013) also discusses the relevance of the profitability factor in pricing the cross-section of average stock returns.

Recent empirical evidence supports the presence of funding liquidity across a wide range of securities. Frazzini and Pedersen (2014) show that leverage constraints are strong and significantly reflected in the return differential between leveraged low-beta stocks and de-leveraged high-beta stocks. These authors argue that the positive and highly significant risk-adjusted returns relative to traditional asset pricing models shown by portfolios sorted by the level of market beta are explained by shadow cost-of-borrowing constraints.[9] The authors illustrate their argument by proposing a market neutral BAB factor consisting of the difference between long levered low-beta stocks and short de-levered high-beta securities. The authors provide convincing evidence that the BAB factor generates high and consistent performance in each of the major global markets and asset classes, and that the results are independent of the asset pricing model employed in the analysis of performance. The BAB factor is downloaded from AQR Capital Management Database. We also employed the market-wide illiquidity factor of Pastor and Stambaugh (2003), which is obtained from Lubos Pastor's website (http://faculty.chicagobooth.edu/lubos.pastor/research/).

We define the default premium (DEF) as the difference between Moody's yield on Baa corporate bonds and the 10-year government bond yield. Both yields are downloaded from the Federal Reserve Statistical Release.

Finally, we estimate the variance risk premium (VRP) for the S&P 100 Index as the logarithm of the ratio between the realized variance and the risk-neutral variance on the index. The estimation details of risk-neutral variances for both individual stocks and the market are in the next section.

[9] See also the paper of Asness, Frazzini, Gormsen, and Pedersen (2017) for additional evidence supporting this argument.

4. Estimation and Descriptive Statistics of (Lower Bound) Expected Risk Premia

We follow Martin (2013, 2017) to estimate risk-neutral variances and (lower bound) expected risk premia. Martin (2013) argues that, under stress market conditions, like October 1987 and the fall of 2008, there is no known way to replicate the payoff of a variance swap. This may be particularly severe for individual stocks, which may experience more frequent and larger jumps than market indices. Martin (2013) proposes the "simple variance swap", which can be hedged at discrete points even if the underlying's asset price jumps. This author develops a risk-neutral variance as an equally-weighted portfolio of options rather than a portfolio of options weighted by the inverse of the square of their strike price, and proposes SVIX, as an alternative to VIX.[10] Martin (2017) shows that the risk-neutral variance is given by

$$Var_{jt,t+\tau}^{Q} = \frac{2R_{ft,t+\tau}}{S_{jt}^{2}}\left[\int_{0}^{F_{jt,t+\tau}} P_{jt,t+\tau}(K)\,dK + \int_{F_{jt,t+\tau}}^{\infty} C_{jt,t+\tau}(K)\,dK\right], \qquad (9)$$

where $P_{jt,t+\tau}(K)$ and $C_{jt,t+\tau}(K)$ are the prices at time t of τ-maturity put and call options with strike K on either an asset or an index j with price S_{jt}, and $F_{jt,t+\tau}$ is the price of a future contract on the asset with the same maturity such that

$$F_{jt,t+\tau} = R_{ft,t+\tau}\left(S_{jt} - d_{jt}\right), \qquad (10)$$

and d_{jt} represents the present value of dividends paid during the life of the contract.

[10] It is important to note that we must derive (lower bound) expected excess returns from risk-neutral variances. As Martin (2013, 2017) points out VIX estimates risk-neutral entropy rather than risk-neutral variance. In any case, in the Appendix at the end of the paper, we perform a final analysis to compare both procedures.

We approximate expression (9) following the same steps carried out by Jiang and Tian (2005) to solve for their model free implied variance. Thus, we approximate the integrals of expression (9) by the following sums over a finite number of strikes:

$$I_x = \sum_{h=1}^{m}\left[g_{jt,t+\tau}\left(K_h^x\right) + g_{jt,t+\tau}\left(K_{h-1}^x\right)\right] \Delta K^x \;,\; x = C, P, \qquad (11)$$

where m equals 100, and ΔK and $g_{jt,t+\tau}$ are given by

$$\Delta K^x = \frac{\left(K_{\max}^x - K_{\min}^x\right)}{m} \;,\; K_h^x = K_{\min}^x + h\Delta K^x;\; h = 1,\ldots,m \qquad (12)$$

$$g_{jt,t+\tau}\left(K_h^x\right) = \begin{cases} C_{jt,t+\tau}\left(K_h^x\right), x = C \\ P_{jt,t+\tau}\left(K_h^x\right), x = P \end{cases} \qquad (13)$$

For each time-to-maturity τ from six to 60 days, we calculate the risk-neutral variance each day for each underlying asset that has at least three available options outstanding, using all the available options at time t.[11] For the risk-free rate, we use the T-bill rate of appropriate maturity (interpolated when necessary) from OptionMetrics, namely, the zero-coupon curve. For the dividend rate for the index, we employ the daily data on the index dividend yield from OptionMetrics. To infer the continuously compounded dividend rate for each individual asset, we combine the forward price with the spot rate used for the forward price calculations. We obtain the mean continuously compounded dividend rate by averaging the implied OptionMetrics dividends.

[11] The window from six days to 60 days corresponds to the maximum range of time to maturity we allow in the necessary interpolation to have enough options every day in the sample with 30 days to maturity.

In practice, we only observe options at some finite sample set of strikes. We transform the prices of listed options into implied volatilities using the Black and Scholes (1973) model, and we fit a smooth function to the implied volatilities using cubic splines. Then, we extract implied volatilities at strikes K_h^x from the fitted function. Finally, we employ equation (11) to calculate the risk-neutral variance using the extracted out-of-the-money option prices. At each time *t*, we focus on a 30-day horizon maturity, interpolated when necessary following the procedure of Carr and Wu (2009).

We also calculate the market variance risk premium for each day in the sample. We first estimate the realized market variance over the same period as that for which risk-neutral variance is obtained for that day:

$$RV_{mt,t+\tau} = \frac{1}{\tau}\sum_{s=1}^{\tau} R_{mt+s}^2 , \tag{14}$$

where $RV_{mt,t+\tau}$ is realized market variance. As in Carr and Wu (2009), we define the market variance risk premium as,

$$VRP_{mt,t+\tau} = \ln\left(\frac{RV_{mt,t+\tau}}{Var_{mt,t+\tau}^Q}\right). \tag{15}$$

Once we have a time-series of daily risk-neutral variances for each asset and the market from equation (9), we calculate the average risk-neutral variance across all days in each month for every available asset and compute the lower bound of expected excess returns following equation (6). Next, we construct 20 risk-neutral variance-sorted portfolios including approximately the same number of assets in each portfolio. Portfolio 1 (P1) contains the assets with the lowest risk-neutral variance (and thus with the lowest expected return), while Portfolio 20 (P20) includes the stocks with the highest risk-neutral variance. The lower bound of the expected excess return and the realized excess return for each portfolio are computed imposing equal weights for the individual assets within the portfolio.

Table 1 contains the descriptive statistics for the (lower bound) expected excess return or expected risk premia (ERP hereafter) for each of the 20 portfolios and the market. The first column shows that the average ERP goes from 0.56% for P1 to 5.87% for P20. The average (lower bound) expected market risk premium is 0.53% (6.36% on annual basis). Note that the average (lower bound) ERP across portfolios is higher than that for the market, which follows from Jensen´s inequality. The volatility of ERP maintains the same monotonic cross-sectional increase that we observe for the mean. Thus, P1 presents the lowest volatility of expected returns and P20 the highest volatility. Moreover, in the third column of Table 1, we observe that the sensitivity of the (lower bound) ERP of each portfolio with respect to the market ERP increases monotonically with the level of the risk-neutral variance. In the fourth column of Table 1, we show that the realized return market betas of the 20 portfolios increase almost monotonically with the lower bounds. Portfolio P1, with the lowest risk-neutral variance, has the lowest average market beta of 0.47, and portfolio P20 has the highest average beta of 2.07.

Next, we estimate the deltas of Kadan and Tang (2016) for each of the 20 portfolios according to expression (8). We must check whether they satisfy the sufficient condition under which the NCC holds for every portfolio. The last column of Table 1 contains the average deltas of the 20 portfolios estimated with monthly data and the full sample period. The lowest delta is 1.19 for portfolio P1 whereas the highest deltas correspond to portfolios P19 and P20, which are equal to 2.82 and 4.14, respectively. Unconditionally, as long as the true risk aversion is higher than 4.14, we can safely employ the NCC to extract (lower bound) ERP of all 20 portfolios.[12]

5. The relation between Expected Returns and Future Realized Returns

Before discussing the factor structure of (lower bound) ERP and their time-series and cross-sectional variability that we report in the next sections of the paper, it is important to check whether our estimated expected returns actually contain information about future realized returns.

Figure 1 represents the time-varying (lower bound) ERP for representative portfolios. All portfolios tend to have a strong counter-cyclical behavior with high peaks during bad economic times. As expected, this is especially the case for portfolio P20, which is the one with the highest market beta and the highest average lower bound of expected returns. Of course, this highly sensitive portfolio has a very poor performance during those months in which its expected return is especially high.

Although the time-series behavior of (lower bound) ERP looks very reasonable, we want to analyze whether higher ex ante expected returns are associated with higher ex post realized returns. We follow the forecasting exercise proposed by Jensen et al. (2017). The realized return at the end of each month $t + 1$ should be positively associated with the expectation at the end of month t, and negatively associated with the revision of

[12] The issue is certainly quite subtle. On theoretical grounds, for the NCC condition to be justified, we not only need that the inequality holds but the covariance in equation (7) should be constant for all assets. On top of that, NCC the condition must hold conditionally, and not only unconditionally. We estimate rolling deltas with past daily data over the previous 3 months. Of course, deltas are time varying and show numbers higher than 4 for some portfolios and specific months. However, the NCC continues being reasonable. The average percentage of months with delta higher than 4 is as low as 1% for portfolios from P1 to P16, and 5.6% for portfolios P17 and P18. This percentage is higher for P19 and P10 but the maximum value for delta is 6.05 and 10.38, respectively. Note that in Section 9, we discuss the empirical results under the exact condition.

expectations at the end of the month. Therefore, for each portfolio we run the following regression:

$$R^e_{jt+1} = \beta_0 + \beta_1 E_t\left(R^e_{jt+1}\right) + \beta_2 \Delta E_{t+1} + \varepsilon_{t+1}, \tag{16}$$

where R^e_{jt+1} is the realized excess return in month $t + 1$ of asset j, $E_t\left(R^e_{jt+1}\right)$ is the time t (lower bound) expected risk premium of asset j, and ΔE_{t+1} represents the contemporaneous unpredictable innovation in the conditional expected excess return. We assume an AR(1)-process to estimate the predictable component. Therefore, ΔE_{t+1} is given by

$$\Delta E_{t+1} = E_{t+1}\left(R^e_{jt+2}\right) - E_t\left(E_{t+1}\left(R^e_{jt+2}\right)\right) = E_{t+1}\left(R^e_{jt+2}\right) - \left[\rho_0 + \rho_1 E_t\left(R^e_{jt+1}\right)\right]. \tag{17}$$

Thus, in the regression equation (16), the ex-ante (lower bound) expected excess return should be positively associated with the ex post realized excess return, which implies that the β_1 slope coefficient should be positive. An upward contemporaneous revision of expectations should imply a drop in the price (and in realized returns), which suggests a negative β_2 coefficient. Finally, β_0 should be zero.

Panel A of Table 2 contains the results for the full sample, and for five selected individual portfolios, P1, P5, P10, P15, and P20. Standard *t*-stats are in parenthesis, whereas *t*-stats with Newey-West (1987) heteroscedasticity and autocorrelation consistent standard errors are in brackets.

Although we work with individual portfolios, our results resemble the evidence reported by Jensen et al. (2017) for the S&P 500 index. We find that $\hat{\beta}_1$ is insignificant and the predictability is driven by the innovation in the expectation term with a negative and clearly significant slope for all 20 portfolios.[13] The intercept is significantly different from zero for 10 out of 20 portfolios (generally, for the lowest ERP portfolios). The global adjustment of the forecasting model is relatively high, taking into account the difficulties in predicting returns with one-month horizon, and the fact that we work with portfolios rather than with the market. The *R*-square is higher than 11% for 15 portfolios, showing a maximum of 21.6% for P17.[14]

Panel B of Table 2 displays the empirical results excluding the NBER recession dates. During recessions, (lower bound) ERP are high, but the ex-post realized returns are very negative. This suggests that the model may work better in the sub-sample without official recession dates. In all five portfolios, the $\hat{\beta}_2$ coefficient remains negative and strongly statistically significant. However, relative to the results reported in Panel A, the $\hat{\beta}_1$ coefficient becomes positive for 15 portfolios and significant for P15.[15]

Now we evaluate the forecasting ability jointly contained in the 20 risk-neutral sorted portfolios. Specifically, we recognize that portfolios with higher expected returns should predict higher future realized returns than portfolios with lower expected returns. With this in mind, we proceed as follows. For each month of our sample period, we form an overall portfolio composed of our original 20 portfolios where the weights are given by the ranking based on expected excess returns:[16]

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[13] Although we show the estimated coefficients for five portfolios, the results are available upon request for all 20 portfolios.

[14] Our results should be compared to those in the last column of Tables 2 and 3 in Jensen et al. (2017) where SVIX is used as expected excess return. It must be noted that they also include ex ante SVIX and VIX as predictors in regression (16). This explains why their *R*-square is higher than our reported *R*-squared.

[15] Jensen et al. (2017) also find that $\hat{\beta}_1$ shows the expected positive sign only when they exclude the Great Recession.

[16] See Asness, Moskowitz, and Pedersen (2013).

$$\omega_{pt} = \frac{rank\left[E_t\left(R^e_{pt+1}\right)\right]}{\sum_{p=1}^{20} rank\left[E_t\left(R^e_{pt+1}\right)\right]}. \tag{18}$$

Given the logic of the empirical exercise, this is a useful procedure because it explicitly avoids short-selling strategies. For this combined portfolio, we compute the expected excess return at each month *t*, the corresponding realized excess return at month *t+1*, and the cumulative realized excess return for 2-, 3-, 6- and 12 month-horizons We then evaluate the forecasting ability of expected returns at the different horizons by running the regression

$$R^e_{pt+\tau} = \beta_0 + \beta_1 E_t\left(R^e_{pt+\tau}\right) + \beta_2 \Delta E_{t+\tau} + \varepsilon_{t+\tau};\ \tau = 1,\ 2,\ 3,\ 6 \text{ and } 12\,. \tag{19}$$

Note that this is an out-of-sample forecasting analysis. Given that we employ options with one-month to maturity, the (lower bound) expected excess returns should significantly predict the future realized excess returns over the shortest horizon. In any case, we also want to explore the information content in near-term expectations for future realized returns in longer horizons.

Table 3 contains the empirical results. At the one-month horizon, all coefficients have the correct theoretical sign. The estimated intercept is insignificantly different from zero, and the $\hat{\beta}_1$ coefficient shows weak predictability. It is positive and marginally significant for the one-month horizon, with *p*-values of 0.075 and 0.078 for the OLS and adjusted *t*-statistics, respectively. Moreover, as expected, $\hat{\beta}_1$ is insignificantly different from zero for the rest of the horizons. The $\hat{\beta}_2$ coefficient is higher than one with the negative desired sign, and highly statistically significant for all horizons up to 6 months. As pointed out before, given the construction of the lower bounds, it is important to notice that the truly relevant forecasting horizon is the shortest one. Finally, *R*-square coefficients indicate generally a good adjustment with maximum forecasting ability for the 3-month horizon.

The overall results imply that expected returns contain relevant information about future one-month realized returns and the reasonable updating or revisions of expectations dominate the forecasting regression over and above the expected returns themselves.

6. The Factor Structure of (Lower Bound) Expected Risk Premia

This section discusses one of the key research question in this paper: What is the factor structure of expected returns?

6.1 Extracting Principal Components

We first extract the principal components from the standard approach, which uses the *N x N* sample variance-covariance matrix of the (lower bound) expected excess returns of our sample of 20 risk-neutral variance-sorted portfolios. In addition, we employ the approach of Connor and Korajczyk (1988). They propose using the eigenvectors associated with the *K* largest eigenvalues of the *T x T* cross product matrix given by

$$\Omega = \frac{1}{N}\left[E_t\left(R_{t+1}^e\right)\right]'\left[E_t\left(R_{t+1}^e\right)\right], \tag{20}$$

where $E_t\left(R_{t+1}^e\right)$ is the 20-dimensional vector of (lower bound) ERP at time *t*. It can be shown that as the cross-section becomes large, the *K x T* matrix with rows consisting of the *K* eigenvectors of Ω will converge to the matrix of factor realizations. Therefore, the estimated *K x T* dimensional matrix represents the excess returns that replicate the realizations of non-observable systematic factors.

Table 4 contains the percentage explained by the first five principal components estimated from both (lower bound) ERP and realized excess returns of our 20 risk-neutral variance-sorted portfolios. Panel A shows the results obtained by the traditional approach. It turns out that the first two principal components of lower bound expected returns explain 99.4% of the variability of expected returns. It seems that two factors may be sufficient to explain the cross-sectional variability of expected returns. On the contrary, the first two principal components of realized returns explain only 76.8% of the variability. The time-varying behavior of the first two principal components of (lower bounds) expected returns during our sample period is displayed in Figure 2. The first principal component, which explains 96.6% of the variability of expected returns, follows closely the counter-cyclical patter of expected returns shown in the previous section of the paper. The second principal component, which only explains an additional 2.8%, tends to be negative during bad economic times and its variability is much lower relative to the first principal component.

Most of the market betas, estimated from a multiple OLS regression of realized returns of each of the 20 portfolios on the two principal components of (lower bound) expected returns, tend to be negative for both principal components. In the case of the first principal component, betas are (almost monotonically) more negative the higher the risk-neutral variance of the portfolio is. The behavior of betas relative to the second principal component is not as smooth as the in the first case. These results are consistent with the strong counter-cyclicality of the first principal component of lower bounds. The principal component moves negatively with respect to realized returns and positively relative to expected returns.

Panel B of Table 4 displays the results using the principal components estimated with the Connor and Korajczyk (1988) procedure. The results are very similar to the ones obtained under the first approach. In this case, the first two principal components explain 99.8% of the variability of lower bounds. The first component explains a slightly higher variability reaching 98.7% instead of 96.6% of the first approach. In any case, it seems clear that two principal components are enough to capture most of the variability of (lower bound) expected risk premia. The percentage of realized returns explained by the first five principal components is practically equal to the percentage obtained by the traditional procedure.

6.2 The Time-Series Behavior of the Two Principal Components of (Lower Bound) Expected Risk Premia

Next, we address the key issue of understanding the underlying risk factors that explain the temporal behavior of these two first principal components. We employ the components estimated under the traditional approach. We select a full battery of candidates that have been shown to have explanatory power in the time-series and cross-sectional variability of returns in previous literature. Panel A of Table 5 shows the time-series determinants of the first principal component. Below the regression-estimated coefficients, we report in parentheses the *t*-statistic based on the traditional OLS standard errors, and in brackets the *t*-statistic based on HAC standard errors. We first analyze the explanatory capacity of the FF five factor risks employed in their five-factor model. It turns out that, the excess market return and the HML factor move negatively with the first principal component, but they lose statistical significance when we employ HAC standard errors. When we add the MOM factor, all FF factors become statistically significant except SMB. The adjusted *R*-square is 12%. Then, we analyze the individual explanatory power of the QMJ and BAB factors, the market-wide illiquidity factor, the DEF premium, and the market variance risk premium (VRP). All of them are statistically significant, and the DEF premium by itself has an *R*-square of 44.1%, which is much higher than the *R*-

square of the augmented (with MOM) FF five-factor model. The next regression includes all variables together. Neither the MOM factor, nor the FF factors are statistically significant. The market-wide illiquidity is also estimated with noise. However, the estimates of the QMJ and the BAB factors, and the DEF premium are statistically different from zero with relatively high HAC *t*-statistics. The market VRP is statistical significant but it losses significance when we employ HAC standard errors. We obtain similar results when we run regressions with these four factor plus the market excess return, which presents a non-statistical significant negative coefficient. When we drop the market portfolio, all factors significantly explain the behavior of the first principal component, as we estimate the market VRP with more precision.

The positive relation between the first principal component of (lower bound) ERP and the QMJ, DEF premium and market VRP suggests that the variables tend to be high in bad economic times. Indeed, Asness et al. (2014) show that the QMJ factor displays large realized returns during downturns, which indicates that the quality-based factor does not exhibit bad-times risk. In particular, they plot the risk-adjusted returns of the QMJ factor against market excess returns and show that the quality factor presents a mild positive convexity, which suggests that the QMJ factor benefits from flight-to-quality during financial and economic crises. The BAB factor, which may be a proxy for funding liquidity, presents a negative and significant relation with the first principal component. Note that the way in which Frazzini and Pedersen (2014) construct the BAB factor implies that low or negative returns of this factor are times of poor funding liquidity or high borrowing constraints. The adjusted *R*-square of the four-factor model is 53.4%, and the inclusion of the excess market return practically has no effect on the adjusted *R*-square.[17]

[17] Given the strong effect of the DEF premium, we analyze the explanatory power of the credit risk premium of Asvanunt and Richardson (2016), which can also be downloaded from the AQR data library. The credit risk premium is the long-term corporate bond total return minus empirical-duration-matched long-term

Panel B of Table 5 shows the results regarding the second principal component. This second factor is much harder to explain than the first component. The relative smooth behavior of the second principal component may be an explanation of this finding. If anything, the HML, CMA, MOM, the market-wide illiquidity factor, and the market VRP with a negative sign show a very weak statistical significance. Overall, the adjusted *R*-square is low and equal to 6.1%. To be precise, across alternative combinations of factors, the MOM factor is the only variable statistically different from zero, even with respect to HAC standard errors. Thus, there is a positive and significant relation between the second principal component of expected returns and the MOM factor.

7. The Cross-Sectional Variability of (Lower Bound) Expected Risk Premia

Having documented the time-series determinants of the first two principal components of lower bounds, we now turn to study the variability of expected excess returns in the cross-section. Our approach is to perform a traditional two-pass cross-sectional regression of Fama and MacBeth (1973) with monthly data, and the estimated (lower bound) ERP of the 20 risk-neutral variance-sorted portfolios as the left-hand side variable. We define the explanatory aggregate factors in the following subsections.[18] We also employ the rigorous econometric methodology of Kan, Robotti, and Shanken (2013) (KRS hereafter), who derive the asymptotic distribution of the cross-sectional *R*-square as a measure of model ability to price the cross-section of average returns. Moreover, they provide standard errors of risk premium estimators adjusted for errors-in-variable and model misspecification.

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government bond total return. Using the available until December 2014, this variable presents a negative and significant relation with the first principal component even if we control for the DEF premium. When we run the model with the four significant state variables, the credit risk premium remains statistically different from zero only with OLS standard errors, but not with HAC standard errors.

[18] Kadan and Tang (2016) also perform a cross-sectional analysis with stock characteristics as explanatory variables.

7.1 The Cross-Sectional of Principal Components

Our first cross-sectional test performs the following cross-sectional regression:

$$E_t\left(R^e_{pt+1}\right) = \lambda_0 + \lambda_1 \beta_{p,f1} + \lambda_2 \beta_{p,f2} + e_{pt} \ ; \ p = 1,\ldots,20 \ , \tag{21}$$

where $E_t\left(R^e_{pt+1}\right)$ is the (lower bound) expected risk premium of portfolio *p*, and the two betas for each portfolio are estimated using rolling time-series regressions of the observed returns of each portfolio on the two principal components of lower bounds (f_1 and f_2), using the past 59 months and the current month:[19]

$$R^e_{pt} = \alpha_p + \beta_{p,f1} f_{1t} + \beta_{p,f2} f_{2t} + \varepsilon_{pt} \ . \tag{22}$$

Panel A of Table 6 contains the results using the (lower bound) expected risk premium of each portfolio as test assets. Below the risk premium estimators, we report the *p*-values associated with the traditional Fama and MacBeth (1973) standard error in parentheses and, in brackets, the *p*-values of the standard errors adjusted for errors-in-variable and the potential misspecification of the model due to KRS (2013). We provide two *R*-square statistics as measures of goodness of model fit. The first number is the standard cross-sectional *R*-square given by the following expression:

$$R^2 = 1 - \frac{Var_N\left(\bar{\hat{e}}_p\right)}{Var_N\left(\overline{E\left(R^e_p\right)}\right)}, \tag{23}$$

where $Var_N(\cdot)$ indicates cross-sectional variance across all *N* (20) portfolios, $\bar{\hat{e}}_p$ is the average estimated pricing error of portfolio *p*, and $\overline{E\left(R^e_p\right)}$ is the average of the (lower

[19] In all cross-sectional regressions reported in the next sections, we estimate betas using the same rolling window with 60 months data of realized returns on the risk factors.

bound) expected risk premia of portfolio *p*. The second cross-sectional statistic is the *R*-squared suggested by KRS (2013) and below, in parenthesis, we provide the *p*-value for the null that the estimated *R*-square equals zero.

According to the classic standard errors, the two risk premia associated with each principal component are strongly statistically significant. This is also true for the first principal component even if we adjust the standard error. However, in the case of the second principal component, the adjusted standard error presents an associated *p*-value, which is high and equal to 0.42. As expected, the sign of the first risk premium is negative given the counter-cyclical pattern of the first principal component. This first factor increases in times of high marginal utility, which suggest a negative risk premium. This is exactly what we find in the cross-sectional results. On the other hand, the risk premium associated with the second principal component is positive and much lower than the first one. These betas explain approximately 91% of the cross-sectional variability of expected returns. Figure 3.1 visualizes the strong cross-sectional fit reflected in the high *R*-square. The most problematic portfolio is P20 with a high pricing error of 1.01%. The high variability of this portfolio explains that the cross-sectional *R*-square due to KRS (2013) is lower than the traditional cross-sectional *R*-square, and equal to 52%. This statistic is one minus the square of the pricing errors weighted by the inverse of the variance-covariance matrix of returns, and thus it assigns a much larger weight to portfolio 20. In any case, the *p*-value is low and equal to 0.06.

Panel B of Table 6 reports the results of the cross-sectional regression with average realized excess returns as the dependent variables on the betas of the principal components extracted from the (lower bound) expected returns. None of the estimated risk premium coefficients is statistically different from zero and the classic cross-sectional *R*-square is 24%. Finally, Panel C shows the results of the cross-section of average realized excess returns on the betas of principal components obtained from the variance-covariance matrix of realized excess returns. Note that this is the usual empirical procedure of testing asset pricing models. As before, the risk premia are not statistically different from zero, and the classic *R*-square is still low and equal to 10.5%. The poor adjustment of the model can be visualized in Figure 3.2.

7.2 A Five-Factor Multi-Beta Pricing Model

In Section 6 of this paper, we show that the return generating process underlying the first principal component of (lower bound) ERP may be written as

$$f_{1t} = \alpha + \beta_1 R^e_{mt} + \beta_2 QMJ_t + \beta_3 BAB_t + \beta_4 DEF_t + \beta_5 VRP_{mt} + \varepsilon_t \quad , \tag{24}$$

where R^e_m is the excess market portfolio returns, and other factors have already been described in Section 6. This specification explains 53.5% of the time-series variability of the first principal component (see Table 5). Thus, when analyzing the cross-sectional variability of expected returns, we employ an ICAPM 5-factor model, which is consistent with the first principal component generating process,

$$E_t\left(R^e_{pt+1}\right) = \lambda_0 + \lambda_1 \beta_{p,excm} + \lambda_2 \beta_{p,qmj} + \lambda_3 \beta_{p,bab} + \lambda_4 \beta_{p,def} + \lambda_5 \beta_{p,vrp} + e_{pt} \;;\; p = 1,\ldots,20 \tag{25}$$

Panel A of Table 7 shows the results employing expected excess returns as test assets. The performance of the model is striking. The cross-sectional *R*-square is 98.2%. The corrected KRS *R*-square is slightly lower but equal to a high 83.6% and it is statistically different from zero. Figure 4.1 displays the clear strong fit between ERP across portfolios and the corresponding fitted values.[20] Again, the portfolio P20 presents

[20] In a recent paper, González-Urteaga and Rubio (2016) deals with the cross-sectional variability of volatility risk premia. This is very different from this paper, even though default is a key aggregate risk

the highest pricing error but it is equal to 0.47% which is a much lower error than in the case of principal components.

The market risk premium is positive and strongly significant. As we report later, this contrasts with the negative market risk premium obtained when we use average realized returns. The risk premia associated with the QMJ, DEF, and market VRP are statistically different from zero with the right negative sign. This holds for both, the classic and KRS standard errors. Note that all these three factors have a negative correlation with the excess market return. This implies that they have positive values in high marginal utility times, which explains the negative and significant risk premium associated with these factors. The risk premium of the BAB factor is also negative, but it is statistically different from zero only when we employ the classic standard errors. The *p*-value becomes 0.34 for the KRS standard errors. It turns out that the betas of high (lower bound) expected returns, like portfolios P18 to P20, are highly negative with respect to the BAB factor, and positive with respect to portfolios P1 and P2. It turns out that this also holds for the QMJ factor and the default premium. The performance of portfolios P18 to P20 becomes worse when QMJ, BAB, and the default premium increase. This may explain the overall BAB negative risk premium, although it is being estimated with much more noise.

The pricing of the DEF premium and the market VRP deserves a more detailed comment. First, González-Urteaga and Rubio (2016) show that the DEF premium is a key factor explaining the cross-sectional variability of the volatility risk premia. They also show that this result reflects a very different behavior of the underlying components of their sample portfolios with respect to credit risk that generates a significant dispersion

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factor in both papers. Our paper is concerned with the risk premium of expected excess returns. Their paper is related to the risk premium of volatility.

of the volatility swap pricing of their portfolios. In our case, portfolio P20 has a high and negative return beta relative to the DEF premium. This suggests that the underlying components of this portfolio have a high credit risk relative to the rest of the portfolios used in our sample, and investors are willing to pay a high variance swap price to hedge default risk. Second, González-Urteaga and Rubio (2017) show that the DEF premium and the market VRP are priced economically and statistically different in the volatility and return segments of the market. On average, common factors in both segments explain 90% of the variability of volatility risk premium portfolios, but only 65% of the variability of equity return portfolios. Indeed, the market VRP is priced significantly in the volatility segment but not in the equity portfolios. In this paper, we find that both factors, DEF premium and market VRP, are priced in the cross-section of expected excess returns. Note that the test assets are expected returns extracted from the volatility segment of the market. In this sense, this new evidence is consistent with the findings of González-Urteaga and Rubio (2017).[21]

Panel B of Table 7 shows the empirical results regarding average realized returns. The nice cross-sectional fit of expected returns strongly contrasts with the much poorer fit of the model when we employ average realized returns. Figure 4.2 illustrates this weak cross-sectional fit. None of the risk premia is statistically different from zero, and the traditional cross-sectional *R*-square is 35.9%. The intercept of the model is highly positive and equal to 9.96% on annual basis. This is not an economically sensible magnitude for the zero-beta return, and it is much higher than the reasonable intercept in Panel A, which equals 3.0% on annual basis.

[21] See also the evidence of Barras and Malkhozov (2016) who rejects the null hypothesis that the conditional market VRP measured in the equity and option markets are equal.

The consistent significant behavior of the model both, in the time-series and the cross-section makes this five-factor model to be a robust and clarifying model for explaining (lower bound) expected excess returns.

8. Empirical Results with an Alternative Portfolio Sort

We now provide additional evidence about (lower bound) ERP by constructing portfolios using market betas of realized returns of individual assets. More specifically, using daily returns for a given month, we estimate market betas for all stocks in the sample. We rank the stocks according to the market beta for any given month *t*, and form 20 market beta-sorted portfolios during the following month. Then, given the stock components in all 20 portfolios during each month, we estimate the risk-neutral variance and the (lower bound) expected excess return for each of the 20 portfolios for all months in the sample. Portfolio 1 (P1) contains the assets with the lowest market beta, and portfolio 20 (P20) includes stocks with the highest beta. The (lower bound) ERP and the realized return for each portfolio are computed using equal weights for the individual assets within the portfolio.

Table 8 contains the descriptive statistics for the (lower bound) ERP for each of the 20 portfolios. As in the previous sorting, based on risk-neutral variances, portfolios with low market betas tend to have lower ERP, lower volatility, and lower betas relative to the expected market return portfolio. On the other hand, portfolios with high market betas have higher (lower bound) expected excess returns, higher volatility, and higher expected return betas. However, contrary to the previous ranking shown in Table 1, the relation is not monotonic. Portfolio P1 has a relatively high (lower bound) ERP. The (lower bound) expected excess returns start increasing in a monotonic way only from portfolio P3 onwards. A similar finding is obtained for the volatility and market betas of (lower bound) expected excess returns. Note that, as in Table 1, the betas of column three of Table 8 are estimated with respect to the (lower bound) market ERP. Overall, the cross-sectional differences for all three statistics are lower when we rank by market betas than when we rank by risk-neutral variances. The last column shows a very close pattern between market betas of realized returns and (lower bound) ERP. The betas are higher the higher the (lower bound) ERP is. As before, the relation is not monotonic with a relatively high beta for portfolio P1.

Rather than presenting all the evidence reported for the 20 risk-neutral variance-sorted portfolios, we just concentrate on the cross-sectional variability of (lower bound) ERP using the five-factor model described in Section 7.2. Panel A of Table 9 displays the results for the (lower bound) expected excess returns. The results are highly significant, although they are not as impressive as in Table 7. The KRS *R*-square is 66.4% instead of 83.6% in Table 8, and is also statistically different from zero. All risk premia show the expected sign with similar magnitudes to the ones reported in Table 7. The beta of the BAB factor is now negative and statistically different from zero, even with respect to the adjusted standard error of KRS. The main difference with respect to Table 7 is that the default beta remains negative but it is not statistically significant. Default beta is a key explanatory variable when portfolios are sorted by risk-neutral variance but it losses statistical significance when we sort by stock market betas. As already pointed out, González-Urteaga and Rubio (2016) show that default is a very relevant factor to explain the cross-sectional pricing of assets with high credit risk, which seems to be embedded in sorts based on risk-neutral variances or volatility risk premia. However, when we sort assets by market betas, it seems that the economic risk component dominates the financial risk component of market betas. Hence, default beta losses explanatory capacity of the cross-section of (lower bound) of expected excess returns. This argument is consistent with the lower cross-sectional dispersion of (lower bound) expected excess returns reported in Panel A Table 9 relative to the dispersion displayed in Panel A of Table 1.

Finally, Panel B of Table 9 contains the results regarding average realized returns. Given the lack of cross-sectional dispersion found in the realized returns of the 20 portfolios sorted by market beta, it is not surprising to find very poor results when trying to explain the average cross-sectional difference with the five-factor model. The KRS *R*-square is not statistically different from zero.

9. The Expected Risk Premia under the Equality Condition: Empirical Results and the Relative Effects of Potential Biases

We express the expected risk premium of any asset *j* as,

$$E_t^P\left(R_{jt+1}\right)-R_{ft}=\frac{1}{R_{ft}}Var_t^Q\left(R_{jt+1}\right)-Cov_t^P\left(M_{t+1}R_{jt+1},R_{jt+1}\right). \qquad (26)$$

Given that the SDF is not observable, we cannot calculate the covariance in the right-hand side of the expression unless we impose an asset pricing model. This is why we work with the lower bounds of ERP. However, it turns out that Ghosh et al. (2016 a, b) propose a non-parametric estimation of an out-of-sample SDF which only depends on asset return. It is known as the Information Stochastic Discount Factor (ISDF).

9.1. The Information Stochastic Discount Factor

The basic idea to obtain the ISDF is to minimize the relative entropy of the risk-neutral measure with respect to the physical measure. It turns out that this can be done by the following maximization problem for $\bar{M}=1$

$$\underset{\{M_t\}_{t=1}^T}{arg\,min}\frac{1}{T}\sum_{t=1}^{T}M_t\,lnM_t \quad s.t.\frac{1}{T}\sum_{t=1}^{T}M_tR_t^e=0\ , \qquad (27)$$

where M_t is the SDF that prices a given set of asset returns at time *t*, and R_t^e is an *N*-vector of excess returns over the risk-free rate. Ghosh et al. (2016 a, b) argue that the solution can be obtained by the corresponding duality

$$\hat{M}_t \equiv M_t\left(\hat{\theta}_T, R_t^e\right) = \frac{e^{\hat{\theta}_T' R_t^e}}{\sum_{t=1}^{T} e^{\hat{\theta}_T' R_t^e}} \quad , \tag{28}$$

where $\hat{\theta}$ is the vector of Lagrange multipliers that solve the unconstrained convex problem

$$\hat{\theta}_T := \underset{\theta}{arg\,min} \sum_{t=1}^{T} e^{\hat{\theta}_T' R_t^e} \quad , \tag{29}$$

which is the dual formulation of the entropy minimization problem. Note that the normalization $\bar{M} = 1$ produces the demeaned SDF. In order to obtain the monthly ISDF for an economically reasonable magnitude, we actually employ the following expression:

$$\hat{M}_t \equiv M_t\left(\hat{\theta}_T, R_t^e\right) = \frac{T}{R_{ft}} \frac{e^{\hat{\theta}_T' R_t^e}}{\sum_{t=1}^{T} e^{\hat{\theta}_T' R_t^e}} \quad , \forall t \, . \tag{30}$$

We follow the out-of-sample rolling estimation procedure suggested by Ghosh et al. (2016 b). We employ daily data of 60 portfolios, including the 25 FF portfolios by size and investment aggressiveness, the 25 FF portfolios by book-to-market and profitability and the 10 portfolios sorted by momentum.[22] At the end of each year, we use a rolling window of previous 30 years of daily data to estimate $\hat{\theta}_T$. These parameters remain

[22] Given that the values for ISDF depend on the returns of the assets employed in its estimation, we decided to use portfolios based on all factors of the FF five-factor model plus momentum in order to capture as many differential characteristics as possible.

constant to compute daily values for the ISDF in the next year. Then, the window rolls one year to generate the complete daily series of ISDF. The covariance in the right-hand side of expression (26) is estimated monthly using daily data of the ISDF and realized returns within the month.

9.2 The Empirical Results under Exact Expected Risk Premia

Table 10 reproduces the factor structure of expected returns but with the exact condition. The percentage explained by the first principal component is lower than in the case of lower bounds. This is the case independently of the methodology employed to extract principal components. Thus, the first principal component explains 84.4% of the variability of exact expected excess returns relative to the 96.2% for the case of lower bounds. We need even more than the first five principal components of exact expected returns to explain the 99.4% captured by the first two principal components of lower bounds under the traditional estimation procedure. In any case, as before, these percentages are much higher than in the case of realized returns.

We now turn to the cross-sectional determinants of exact expected excess returns. Panel A of Table 11 shows the results regarding the principal components. The results are very similar to the ones reported in Table 6, although the risk premium associated with the first principal component is estimated with slightly less precision. Panel B of Table 11 shows that the four variables explaining a large percentage of the temporal behavior of the first principal component of lower bounds, QMJ, BAB, DEF premium, and market VRP, are also significant determinants of the first principal component of exact expected returns. As before, all four risk premia are negative. The intercept and the market risk premium are 3.7% and 8.2% on annual basis, respectively. Both coefficients are statistically different from zero and show reasonable magnitudes for both the zero-beta rate and the market risk premium. The variability explained by the five-factor model is practically the same as with lower bounds. Once again, this result suggests that the five-factor model is a powerful and reasonable candidate to explain the cross-sectional behavior of expected excess returns.

9.3 The Bias of the (Lower Bound) Expected Excess Returns

Given the estimation of the out-of-sample ISDF, what do we know about the bias of the lower bound? Is the behavior of the covariance term reasonably constant over time? Unless there are strong economic cycle effects affecting that covariance, the bias might be fairly stable and in that case, the use of lower bounds to approximate expected excess returns would be acceptable.

Figure 5.1 shows the first principal component of the (lower bound) expected ERP of the 20 portfolios and the first principal component when we employ the exact expected returns obtained from equation (26). Both series follow a very similar pattern with the same peaks during bad economic times. It seems, however, that the difference between the two series or, in other words, the bias introduced by the lower bound expression, does not remain constant over the economic cycle. We observe a similar finding in Figure 5.2 where we show the second principal components under the lower bound and exact expressions. This evidence seems to suggest that the effect of the bias may have serious effects on the results.

In order to formally verify this potential distortion, we regress the exact principal components on the lower bound principal components. The regression is given by,

$$Exact\ PC_{t,i} = \alpha + \beta\ Lower\ Bound\ PC_{t,i} + \varepsilon_t;\ i = 1,2\ . \qquad (31)$$

We report the results in Panel A of Table 12. The lower bound first principal component significantly explains the exact first principal component. The slope coefficient is positive and highly significant, and the *R*-square is 36.1%. The second principal component is much more problematic. The lower bound second principal component does not seem to explain the exact second component.

In a second analysis, we regress the biases on the lower bound principal components:

$$Bias\ PC_{t,i} = \alpha + \beta\ Lower\ Bound\ PC_{t,i} + \varepsilon_t;\ i = 1,2\ . \tag{32}$$

Panel B of Table 12 contains the results. In this case, we conclude that the first and second principal components estimated with lower bound significantly explain the biases. For the first component, the slope coefficient is high, positive, and statistically different from zero, and the *R*-square is 66.7%. Although the results are weaker for the second principal component, we may safely conclude that the lower bound second principal component is able to explain the corresponding bias. To complete this last regression, it should be the case that the biases do not explain the exact principal components. Therefore, we perform the following final regression,

$$Exact\ PC_{t,i} = \alpha + \beta\ Bias\ PC_{t,i} + \varepsilon_t;\ i = 1,2\ . \tag{33}$$

Panel C of Table 12 shows that, indeed, this is the case for the first principal component. However, the slope coefficient of the second principal component is negative and significant. The higher the bias, the lower the exact second principal component. It is also true that the *R*-square is relatively low and equal to 10.3%.

To summarize the evidence, the potential bias we have when calculating expected excess returns using a lower bound rather than an exact expression is low and statistically non-significant as long as we employ the first principal component. The evidence is more problematic regarding the second principal components. However, it is important to remember that the first principal component explains most of the variation of expected excess returns. Taking our results together, the distortion we find, when using lower bound rather than exact results, may not be economically relevant. We argue that the empirical results under the lower bound approximation, both in the time-series and the cross-section, provide an important first step in understanding the behavior of expected excess returns using option pricing data.

10. Conclusions

After several decades of intense research using realized past returns to study the behavior of stock returns, we still know very little about the factor structure and cross-sectional variability of expected returns. Merton (1980) already shows how difficult estimating the mean market return is, and he argues that only extending the sample over time we are able to approximate means adequately. Sampling at higher frequencies does not help with precise estimation of mean returns.

This paper partially covers this gap using a combination of option pricing results from Martin (2013, 2017), and insights of the recent SDF literature due to Ghosh et al. (2016 a, b). We estimate the lower bound of ERP, and exact ERP for 20 risk-neutral variance-sorted portfolios using option prices. We find that the factor structure of ERP can be summarized with the two first principal components. The first principal component explains 96.6% (84.4%) of the variability of (lower bound) ERP (exact ERP). This first principal component presents a reasonable and strong counter-cyclical behavior. The second principal explains an additional 2.8% (9.7%) of the variability of (lower bound) ERP (exact ERP). These percentages are clearly higher than the percentages found when employing realized returns.

When we use (lower bound) ERP, we conclude that both the time-series and cross-sectional variability of expected returns are mainly explained by similar risk factors. These aggregate variables are the differences between high and low quality portfolios, the differences between leveraged and deleveraged betas (funding liquidity or the tightness of borrowing constraints), the default premium, and the market variance risk premium. All risk premia are negative and statistically significant. The market risk premium is positive and statistically different from zero. These results do not depend on the inclusion of the covariance term in the expression to obtain exact expected excess returns. Overall, our results suggest that expected returns are time-varying in a strong counter-cyclical way and vary much more than what is usually accepted. The robust identification of the set of factors that significantly explain a very large percentage of their variability is an important step to understand the behavior of expected returns. This is especially relevant given that expected excess returns, and changes in their conditional expectations, contain useful information about future realized returns.

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Table 1. Descriptive Statistics of (Lower Bound) Expected Risk Premia and Realized Returns for 20 Portfolios Sorted by Risk-Neutral Variance: January 1996-July 2015.

	Average ERP	Volatility ERP	Market ERP Beta	Market Beta	Delta
P1	0.0056	0.0037	0.6375	0.4722	1.19
P2	0.0069	0.0046	0.8047	0.5393	1.30
P3	0.0078	0.0053	0.9457	0.5844	1.35
P4	0.0086	0.0059	1.0674	0.6206	1.31
P5	0.0093	0.0063	1.1452	0.6681	1.30
P6	0.0100	0.0067	1.2268	0.8187	1.28
P7	0.0107	0.0071	1.3171	0.8274	1.31
P8	0.0113	0.0076	1.4053	0.7627	1.31
P9	0.0120	0.0081	1.5008	0.9955	1.50
P10	0.0128	0.0085	1.5802	0.9717	1.44
P11	0.0136	0.0091	1.6810	1.0429	1.61
P12	0.0145	0.0097	1.7956	0.9932	1.68
P13	0.0157	0.0106	1.9782	1.1528	1.64
P14	0.0170	0.0117	2.1817	1.0879	1.81
P15	0.0185	0.0128	2.3955	1.1576	1.97
P16	0.0205	0.0143	2.6670	1.1855	1.90
P17	0.0236	0.0171	3.1532	1.2998	2.16
P18	0.0281	0.0211	3.8136	1.5392	2.45
P19	0.0355	0.0268	4.7015	1.6151	2.82
P20	0.0587	0.0470	7.9929	2.0688	4.14
MARKET	0.0053	0.0049	1.0000	1.0000	-

This table presents the descriptive statistics of 20 portfolios sorted by risk-neutral variance, where the market is the Standard & Poor 100 Index. The first two columns show the mean and the volatility of the (lower bound) expected risk premia. The third column is the sensitivity of the expected risk premium of the 20 portfolios to the expected market risk premium, the fourth column contains the market beta of realized returns of the components of the twenty portfolios, and the last column the deltas given by expression (8), estimated with monthly data over the full sample period.

Table 2. The One-Month Ahead Forecasting Performance of (Lower Bound) Expected Risk Premia for Selected Portfolios Sorted by Risk-Neutral Variance: January 1996-July 2015.

Panel A: Full Sample					
	P1	P5	P10	P15	P20
$\hat{\beta}_0$	0.014	0.008	0.009	0.008	0.014
	(3.98)	(2.73)	(1.67)	(1.11)	(1.08)
	[3.68]	[1.87]	[1.50]	[0.80]	[0.99]
$\hat{\beta}_1$	-0.345	0.040	0.189	0.135	-0.015
	(-0.60)	(0.12)	(0.49)	(0.41)	(-0.09)
	[-0.49]	[0.10]	[0.41]	[0.21]	[-0.05]
$\hat{\beta}_2$	-4.865	-6.029	-6.244	-4.056	-2.399
	(-3.72)	(-7.99)	(-9.81)	(-5.53)	(-6.69)
	[-3.61]	[-6.81]	[7.26]	[-5.05]	[-6.08]
R-square	0.058	0.167	0.187	0.117	0.162

Panel B: Sample Excluding Recession Dates (March 2001-November 2001 and August 2008-July 2009)					
	P1	P5	P10	P15	P20
$\hat{\beta}_0$	0.013	0.010	0.007	-0.004	0.028
	(3.61)	(2.31)	(1.14)	(-0.51)	(1.80)
	[3.01]	[2.01]	[0.95]	[-0.46]	[1.68]
$\hat{\beta}_1$	0.328	0.107	0.757	1.161	-0.147
	(0.47)	(0.21)	(1.44)	(2.57)	(-0.55)
	[0.45]	[0.19]	[1.33]	[2.14]	[-0.35]
$\hat{\beta}_2$	-5.193	-7.330	-6.051	-5.502	-2.295
	(-2.44)	(-5.67)	(-4.18)	(-4.63)	(-3.62)
	[-1.92]	[-5.22]	[-4.02]	[-4.54]	[-3.28]
R-square	0.029	0.119	0.088	0.121	0.062

This table presents the intercept and slope estimated coefficients from OLS forecasting regressions of (lower bound) expected risk premia on future realized excess returns, and ex-post innovations in expected returns for a one-month horizon:

$$R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \beta_2 \Delta E_{t+1} + \varepsilon_{pt+1}$$

Results refer to five selected portfolios from the 20 risk-neutral variance-sorted portfolios. We report standard *t*-statistics in parentheses, and corrected *t*-statistics in brackets based on heteroskedastic- and autocorrelation-robust standard errors. Panel A contains the evidence for the full sample period, and Panel B displays the results excluding recession dates (March 2001-November 2001 and August 2008-July 2009).

Table 3. The Forecasting Performance of the Ranked-Based Optimal (Lower Bound) Expected Risk Premium Portfolio Constructed from the 20 Portfolios Sorted by Risk-Neutral Variance: January 1996-July 2015.

Ranked-Based Portfolio	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 6$	$\tau = 12$
$\hat{\beta}_0$	-0.000	0.006	0.013	0.009	0.008
	(-0.03)	(1.45)	(4.14)	(3.12)	(3.41)
	[-0.03]	[1.09]	[2.34]	[2.35]	[1.98]
$\hat{\beta}_1$	0.475	0.125	-0.200	-0.029	0.010
	(1.79)	(0.79)	(-1.74)	(-0.28)	(0.12)
	[1.41]	[0.44]	[-0.72]	[-0.14]	[0.05]
$\hat{\beta}_2$	-1.786	-2.096	-2.072	-1.128	-0.470
	(-4.61)	(-9.04)	(-12.38)	(-7.58)	(-3.99)
	[-3.01]	[-7.31]	[-9.16]	[-7.45]	[-2.06]
R-square	0.086	0.265	0.406	0.203	0.068

This table presents the intercept and slope estimated coefficients from OLS forecasting regressions of (lower bound) expected risk premia on future realized excess returns, and ex-post innovations in expected returns for different τ-month horizons:

$$R^e_{t+\tau} = \beta_0 + \beta_1 E_t\left(R^e_{t+1}\right) + \beta_2 \Delta E_{t+\tau} + \varepsilon_{t+\tau}\ ;\ \tau = 1,\ 2,\ 3,\ 6\ and\ 12$$

We employ a ranked-based optimal portfolio of our 20 risk-neutral variance-sorted portfolios. We report standard *t*-statistics in parentheses, and corrected *t*-statistics in brackets based on heteroskedastic- and autocorrelation-robust standard errors.

Table 4. The Factor Structure of (Lower Bound) Expected Risk Premia for 20 Portfolios Sorted by Risk-Neutral Variance. Percentage Explained by the First Five Principal Components: January 1996-July 2015.

	Panel A: Principal Components		Panel B: Principal Components Connor & Korajczyk	
	(Lower Bound) Expected Risk Premia	Realized Excess Returns	(Lower Bound) Expected Risk Premia	Realized Excess Returns
Factor (PC) 1	96.62	66.74	98.66	67.11
Factor (PC) 2	99.38	76.75	99.77	77.22
Factor (PC) 3	99.71	80.50	99.90	80.90
Factor (PC) 4	99.87	83.47	99.95	83.85
Factor (PC) 5	99.93	85.73	99.97	86.05

The two columns of Panel A show the percentage of the variability of the 20 x 20 variance-covariance matrix of (lower bound) expected and realized excess returns of our 20 portfolios, respectively, explained by the first five principal components. The two columns of Panel B report the same percentages of the variability but of the *T x T* covariance matrix, in this case, computed following the procedure of Connor and Korajczyk (1988).

Table 5. Determinants of the Factor Structure of (Lower Bound) Expected Risk Premia. January 1996-July 2015.

Panel A: Determinants of the First Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.027	-0.081	0.035	-0.130	0.096	0.124							
(20.25)	(-2.34)	(0.78)	(-2.25)	(1.57)	(1.57)							0.074
[10.14]	[-1.50]	[0.85]	[-1.24]	[1.58]	[1.25]							
0.027	-0.103	0.057	-0.194	0.128	0.167	-0.090						
(21.01)	(-3.00)	(1.29)	(-3.29)	(2.13)	(2.14)	(-3.62)						0.120
[10.30]	[-1.83]	[1.44]	[-1.96]	[1.96]	[1.88]	[-2.19]						
0.026							0.158					
(20.63)							(3.91)					0.058
[10.54]							[1.78]					
0.028								-0.102				
(21.44)								(-3.33)				0.041
[10.31]								[-2.21]				
0.027									0.070			
(21.38)									(3.71)			0.052
[10.67]									[3.33]			
-0.012										1.589		
(-3.92)										(13.62)		0.441
[-1.72]										[5.32]		
0.029											0.010	
(20.83)											(3.88)	0.057
[9.61]											[2.90]	
-0.005	-0.020	0.018	0.021	-0.002	0.057	-0.008	0.150	-0.129	0.014	1.375	0.004	
(-1.70)	(-0.66)	(0.54)	(0.44)	(-0.02)	(0.98)	(-0.39)	(2.07)	(-4.43)	(0.98)	(11.54)	(1.96)	0.533
[-0.79]	[-0.64]	[0.50]	[0.42]	[-0.02]	[0.83]	[-0.37]	[2.61]	[-3.56]	[0.79]	[4.90]	[1.26]	
-0.006	-0.034						0.131	-0.118		1.410	0.004	
(-1.99)	(-1.20)						(3.25)	(-4.85)		(12.81)	(2.16)	0.535
[-0.95]	[-1.13]						[2.51]	[-4.13]		[5.70]	[1.40]	
-0.006							0.158	-0.113		1.425	0.005	
(-2.19)							(4.73)	(-4.71)		(13.03)	(2.73)	0.534
[-1.02]							[2.99]	[-4.21]		[5.71]	[1.73]	

Panel B: Determinants of the Second Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.003	-0.003	-0.017	-0.050	-0.001	0.043							
(8.52)	(-0.36)	(-1.54)	(-3.46)	(-0.04)	(2.15)							0.050
[5.49]	[-0.28]	[-1.78]	[-2.37]	[-0.05]	[1.82]							
0003	-0.001	-0.020	-0.043	-0.005	0.038	0.011						
(8.38)	(-0.06)	(-1.76)	(-2.81)	(-0.29)	(1.88)	(1.67)						0.058
[5.44]	[-0.05]	[-1.93]	[-1.99]	[-0.30]	[1.73]	[2.04]						
0.003							0.015					
(8.44)							(1.41)					0.004
[5.89]							[1.37]					
0.003								-0.008				
(8.74)								(-1.04)				0.000
[5.76]								[-0.80]				
0.003									0.009			
(8.78)									(1.79)			0.009
[6.17]									[1.48]			
0.003										-0.003		
(2.88)										(-0.08)		-0.004
[1.22]										[-0.03]		
0.003											-0.001	
(7.38)											(-0.85)	-0.001
[4.54]											[-0.60]	
0.003	-0.003	-0.015	-0.036	-0.003	0.036	0.012	0.011	-0.012	0.006	-0.019	-0.001	
(2.70)	(-0.24)	(-1.31)	(-2.08)	(-0.14)	(1.72)	(1.66)	(0.45)	(-1.13)	(1.26)	(-0.45)	(-1.71)	0.061
[1.26]	[-0.26]	[-1.26]	[-1.27]	[-0.11]	[1.28]	[2.04]	[0.46]	[-0.94]	[1.04]	[-0.18]	[-1.19]	
0.002						0.017			0.009		-0.001	
(6.83)						(2.77)			(1.80)		(-1.73)	0.039
[4.01]						[2.37]			[1.56]		[-1.12]	

This table shows the estimated coefficients of time-series regressions of each of the two first principal components on alternative state variables. The first six variables are the intercept and the Fama and French (2015) factors, MOM is the Momentum Factor of Carhart (1997), QMJ is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), BAB is the betting-against-beta factor of Frazzini and Pedersen (2014), P&S is the Pastor and Stambaugh (2003) illiquidity factor, DEF is the default premium, and VRP is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. OLS *t*-statistics are reported in parenthesis and *t*-statistics based on HAC standard errors in brackets.

Table 6. The Cross-Section of Portfolios sorted by Risk-Neutral Variance and the Factor Structure of (Lower Bound) Expected Risk Premia. January 1996-July 2015.

Panel A: (Lower Bound) Expected Risk Premia and Principal Components from the Variance-Covariance Matrix of (Lower Bound) Expected Risk Premia				
	λ_0	λ_{f_1}	λ_{f_2}	*R*-square
	0.0020	-0.0162	0.0006	0.909
	(0.046)	(0.000)	(0.002)	0.523
	[0.000]	[0.003]	[0.417]	(0.06)
Panel B: Average Realized Excess Returns and Principal Components from the Variance-Covariance Matrix of (Lower Bound) Expected Risk Premia				
	λ_0	λ_{f_1}	λ_{f_2}	*R*-square
	0.0069	-0.0043	0.0009	0.243
	(0.012)	(0.322)	(0.235)	0.010
	[0.001]	[0.249]	[0.365]	(0.96)
Panel C: Average Realized Excess Returns and Principal Components from the Variance-Covariance Matrix of Realized Excess Returns				
	λ_0	λ_{f_1}	λ_{f_2}	*R*-square
	0.0097	-0.0020	-0.0007	0.105
	(0.013)	(0.733)	(0.861)	0.011
	[0.004]	[0.725]	[0.875]	(0.93)

This table reports the risk premia estimates from the two-pass cross-sectional regression of returns on rolling betas associated with the two first principal components. Panel A shows the results using (lower bound) expected risk premia, and Panels B and C employ excess realized returns. Principal components come from the variance-covariance matrix of (lower bound) expected risk premia in Panels A and B and from the variance-covariance matrix of realized excess returns in Panel C. We report the *p*-value in parentheses and the Kan, Robotti, and Shanken (2013) adjusted *p*-value in brackets. The cross-sectional *R*-square reported in the first line is 1 minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. Kan, Robotti, and Shanken (2013) propose the cross-sectional *R*-square reported in the second line, and the corresponding (asymptotically valid) *p*-value is given in parentheses.

Table 7. The Cross-Section of Portfolios sorted by Risk-Neutral Variance and a Multi-Factor Asset Pricing Model. January 1996-July 2015.

Panel A: (Lower Bound) Expected Risk Premia

λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-square
0.0025	0.0129	-0.0137	-0.0035	-0.0022	-0.1234	0.982
(0.002)	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	0.836
[0.229]	[0.000]	[0.000]	[0.338]	[0.000]	[0.000]	(0.00)

Panel B: Average Realized Excess Returns

0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-square
0.0083	-0.0003	0.0011	-9.0e-5	-0.0007	-0.0750	0.359
(0.070)	(0.955)	(0.706)	(0.986)	(0.583)	(0.425)	0.063
[0.085]	[0.957]	[0.762]	[0.991]	[0.740]	[0.630]	(0.99)

This table reports the risk premia estimates from the two-pass cross-sectional regression of returns on rolling betas associated with the five aggregate risk factors: *m* represents the market excess return, *qmj* is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), *bab* denotes the betting-against-beta factor of Frazzini and Pedersen (2014), *def* is the default premium, and *vrp* is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. Panel A shows the results using (lower bound) expected risk premia, and Panel B employs excess realized returns. We report the *p*-value in parentheses and the Kan, Robotti, and Shanken (2013) adjusted *p*-value in brackets. The cross-sectional *R*-square reported in the first line is 1 minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. Kan, Robotti, and Shanken (2013) propose the cross-sectional *R*-square reported in the second line, and the corresponding (asymptotically valid) *p*-value is given in parentheses.

Table 8. Descriptive Statistics of (Lower Bound) Expected Risk Premia and Realized Returns for 20 Portfolios Sorted by Market Betas: January 1996-July 2015.

	Average ERP	Volatility ERP	Market ERP Beta	Market Beta
P1	0.0180	0.0124	1.3322	0,7825
P2	0.0139	0.0111	1.3198	0,5845
P3	0.0121	0.0089	1.1555	0,7013
P4	0.0121	0.0089	1.1497	0,6049
P5	0.0124	0.0086	1.2241	0,8002
P6	0.0131	0.0126	1.8217	0,7224
P7	0.0131	0.0094	1.4045	0,7855
P8	0.0128	0.0096	1.4611	0,8551
P9	0.0134	0.0099	1.7066	0,9195
P10	0.0141	0.0098	1.5546	0,9467
P11	0.0145	0.0119	2.1028	0,9436
P12	0.0148	0.0107	1.8546	0,9413
P13	0.0154	0.0111	1.9750	1,1100
P14	0.0164	0.0135	2.4355	1,1467
P15	0.0170	0.0132	2.4062	1,1898
P16	0.0187	0.0165	3.0044	1,2160
P17	0.0209	0.0193	3.4937	1,3019
P18	0.0234	0.0209	3.6818	1,5239
P19	0.0276	0.0249	4.2689	1,6679
P20	0.0390	0.0386	6.1537	1,7391
MARKET	0.0053	0.0049	1.0000	1.0000

This table presents the descriptive statistics of 20 portfolios sorted by market betas, where the market is the Standard & Poor 100 Index. The first two columns show the mean and the volatility of the (lower bound) expected risk premia. The third column is the sensitivity of the expected risk premium of the 20 portfolios to the expected market risk premium, and the last column contains the market beta of realized returns of the components of the twenty portfolios.

Table 9. The Cross-Section of Portfolios sorted by Market Betas and a Multi-Factor Asset Pricing Model. January 1996-July 2015.

Panel A: (Lower Bound) Expected Risk Premia

λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	*R*-square
0.0047	0.0109	-0.0088	-0.0078	-0.0002	-0.1528	0.936
(0.000)	(0.000)	(0.000)	(0.000)	(0.313)	(0.000)	0.664
[0.014]	[0.000]	[0.001]	[0.005]	[0.718]	[0.000]	(0.00)

Panel B: Average Realized Excess Returns

λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	*R*-square
0.0036	0.0043	0.0027	-0.0002	-0.0006	-0.0115	0.152
(0.514)	(0.466)	(0.376)	(0.959)	(0.436)	(0.882)	0.327
[0.625]	[0.645]	[0.545]	[0.972]	[0.789]	[0.923]	(0.68)

This table reports the risk premia estimates from the two-pass cross-sectional regression of returns on rolling betas associated with the five aggregate risk factors: *m* represents the market excess return, *qmj* is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), *bab* denotes the betting-against-beta factor of Frazzini and Pedersen (2014), *def* is the default premium, and *vrp* is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. Panel A shows the results using (lower bound) expected risk premia, and Panel B employs excess realized returns. We report the *p*-value in parentheses and the Kan, Robotti, and Shanken (2013) adjusted *p*-value in brackets. The cross-sectional *R*-square reported in the first line is 1 minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. Kan, Robotti, and Shanken (2013) propose the cross-sectional *R*-square reported in the second line, and the corresponding (asymptotically valid) *p*-value is given in parentheses.

Table 10. The Factor Structure of Expected Returns under the Exact Condition for 20 Portfolios Sorted by Risk-Neutral Variance. Percentage Explained by the First Five Principal Components: January 1996-July 2015.

	Panel A: Principal Components		Panel B: Principal Components Connor & Korajczyk	
	Exact Expected Risk Premia	Realized Excess Returns	Exact Expected Risk Premia	Realized Excess Returns
Factor (PC) 1	84.36	66.74	94.39	67.11
Factor (PC) 2	94.10	76.75	98.03	77.22
Factor (PC) 3	96.20	80.50	98.73	80.90
Factor (PC) 4	97.19	83.47	99.06	83.85
Factor (PC) 5	98.05	85.73	99.35	86.05

The two columns of Panel A show the percentage of the variability of the 20 x 20 variance-covariance matrix of exact expected and realized excess returns of our 20 portfolios, respectively, explained by the first five principal components. The two columns of Panel B report the same percentages of the variability but of the *T x T* covariance matrix, in this case, computed following the procedure of Connor and Korajczyk (1988).

Table 11. The Cross-Section of Exact Expected Risk Premia. January 1996-July 2015.

Panel A: Principal Components	λ_0	λ_{f_1}	λ_{f_2}				R-square
	0.0048	-0.0007	0.0006				0.847
	(0.000)	(0.012)	(0.000)				0.374
	[0.000]	[0.149]	[0.095]				(0.00)
Panel B: Multi-factor	λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-square
	0.0031	0.0068	-0.0075	-0.0032	-0.0008	-0.0335	0.983
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.027)	0.836
	[0.000]	[0.000]	[0.000]	[0.073]	[0.002]	[0.130]	(0.00)

This table reports the risk premia estimates from the two-pass cross-sectional regression of exact expected excess returns on rolling betas associated with the two principal components (Panel A), and with five aggregate risk factors (Panel B). *m* represents the market excess return, *qmj* is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), *bab* denotes the betting-against-beta factor of Frazzini and Pedersen (2014), *def* is the default premium, and *vrp* is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. We report the *p*-value in parentheses and the Kan, Robotti, and Shanken (2013) adjusted *p*-value in brackets. The cross-sectional *R*-square reported in the first line is 1 minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. Kan, Robotti, and Shanken (2013) propose the cross-sectional *R*-square reported in the second line, and the corresponding (asymptotically valid) *p*-value is given in parentheses.

Table 12. The Effects of Potential Biases on the (Lower Bound) Expected Excess Returns

Panel A: $Exact\ PC_i = \alpha + \beta\ Lower\ Bound\ PC_i + \varepsilon_i\ ;\ i=1,2$

	α	β	*Adj. R*-square
(Lower Bound) Expected Excess Returns PC1	0.0082 (8.09) [3.38]	0.3509 (11.55) [3.07]	0.361
(Lower Bound) Expected Excess Returns PC2	0.0031 (7.85) [3.45]	0.2065 (2.98) [1.36]	0.033

Panel B: $Bias\ PC_i = \alpha + \beta\ Lower\ Bound\ PC_i + \varepsilon_i\ ;\ i=1,2$

	α	β	*Adj. R*-square
(Lower Bound) Expected Excess Returns PC1	-0.0096 (-8.39) [-3.65]	0.7400 (21.60) [5.89]	0.666
(Lower Bound) Expected Excess Returns PC2	-0.0009 (-1.96) [-1.19]	0.6656 (8.41) [2.56]	0.229

Panel C: $Exact\ PC_i = \alpha + \beta\ Bias\ PC_i + \varepsilon_i\ ;\ i=1,2$

	α	β	*Adj. R*-square
(Lower Bound) Expected Excess Returns PC1	0.0172 (19.89) [10.63]	0.0335 (0.80) [0.28]	0.003
(Lower Bound) Expected Excess Returns PC2	0.0039 (11.77) [6.79]	-0.2554 (-5.28) [-2.10]	0.103

This table analyzes the effects of using lower bound principal components instead of principal components estimated from exact expected excess returns. The lower bound principal components are from the variance-covariance matrix of the 20 (lower bound) expected excess returns, while the exact principal components are from the variance-covariance matrix of the 20 exact expected excess returns from expression (22). The biases refer to the second term on the right-hand side of equation (22).

Figure 1. (Lower Bound) Expected Risk Premia for some Representative Portfolios. Monthly Data, January 1996-July 2015.

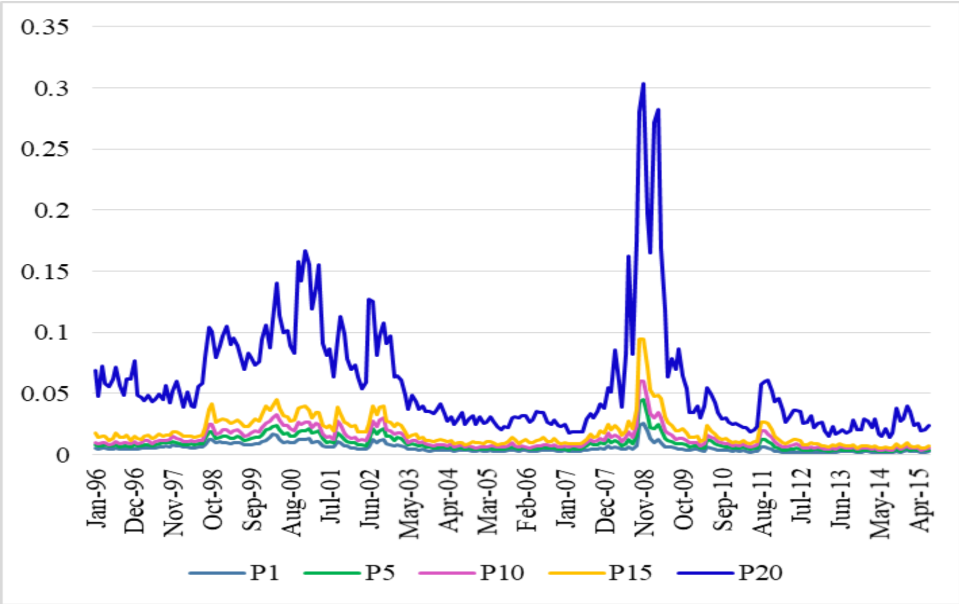


Figure 2. Principal Components from the Variance-Covariance Matrix of (Lower Bound) Expected Excess Returns. January 1996-2015.

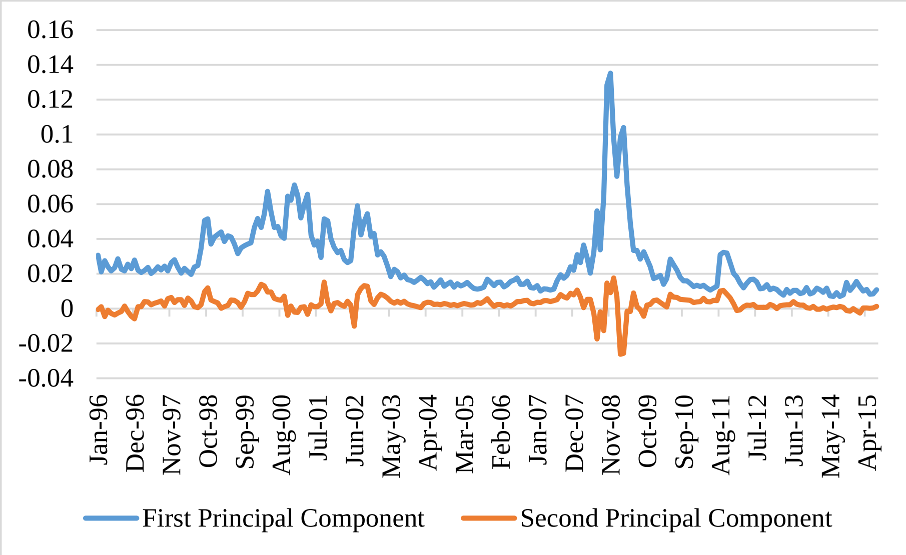


Figure 3.1. The Cross-Section of (Lower Bound) Expected Excess Returns. Two Principal Components Model. January 1996-July 2015.

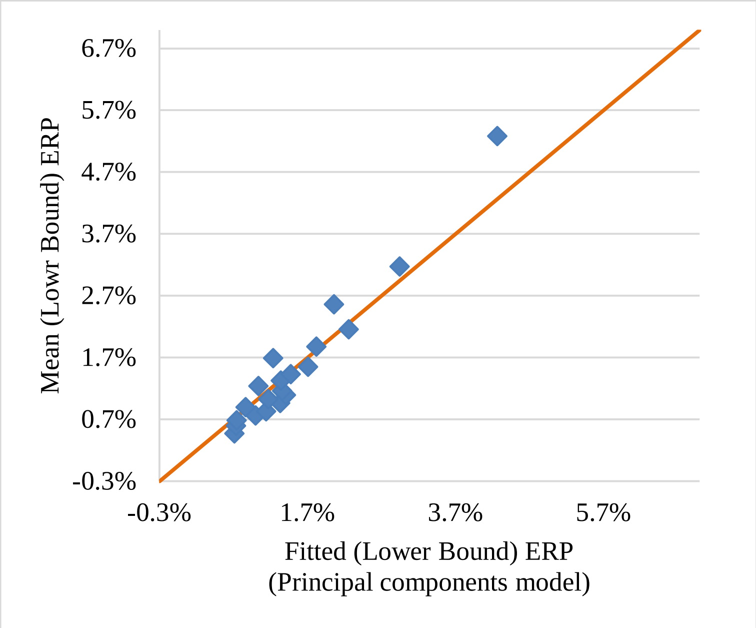


Figure 3.2. The Cross-Section of Average Realized Excess Returns. Two Principal Components. January 1996-July 2015.

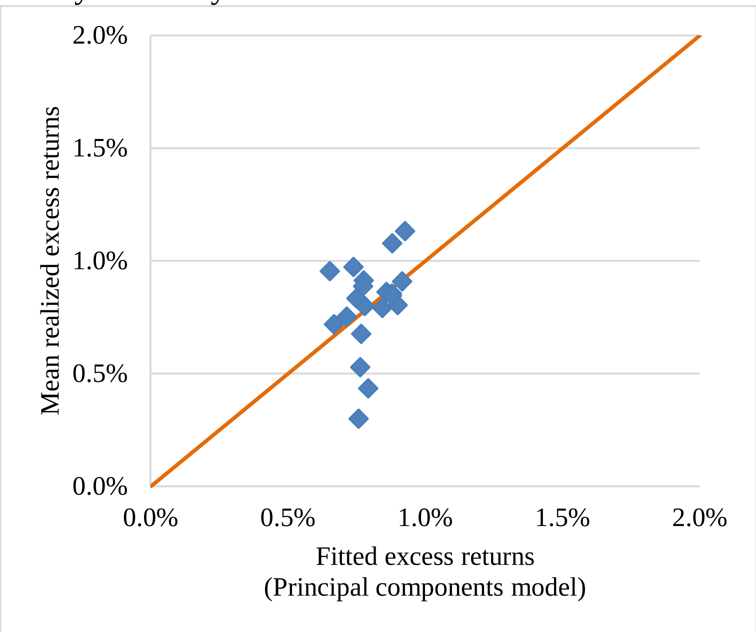


Figure 4.1. The Cross-Section of (Lower Bound) Expected Excess Returns. Multi-Factor Model. January 1996-July 2015.

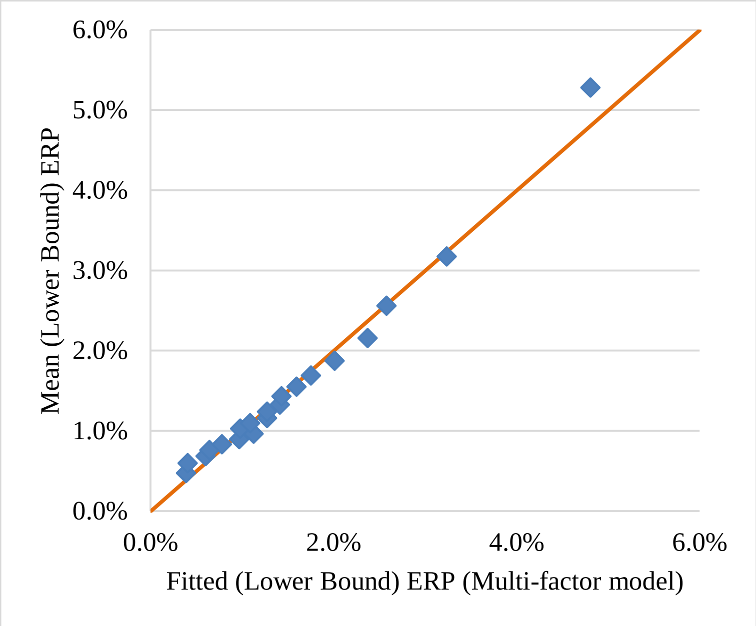


Figure 4.2. The Cross-Section of Average Realized Excess Returns. Multi-Factor Model. January 1996-July 2015.

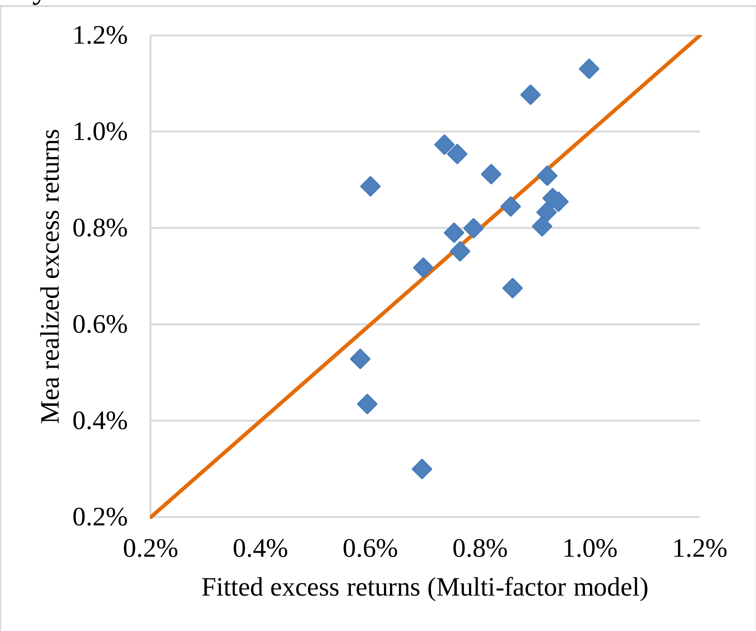


Figure 5.1 Lower Bound and Exact First Principal Components. January 1996-2015.

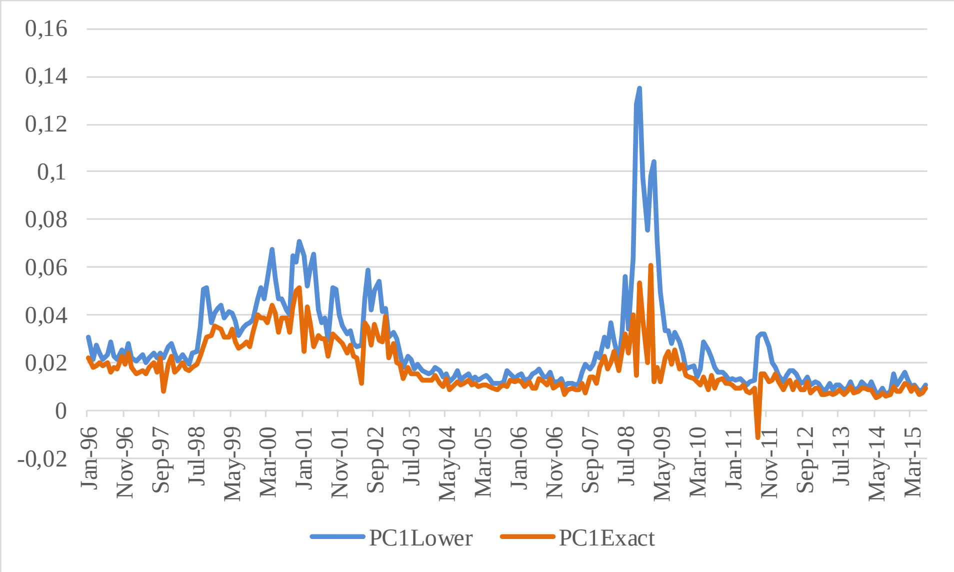
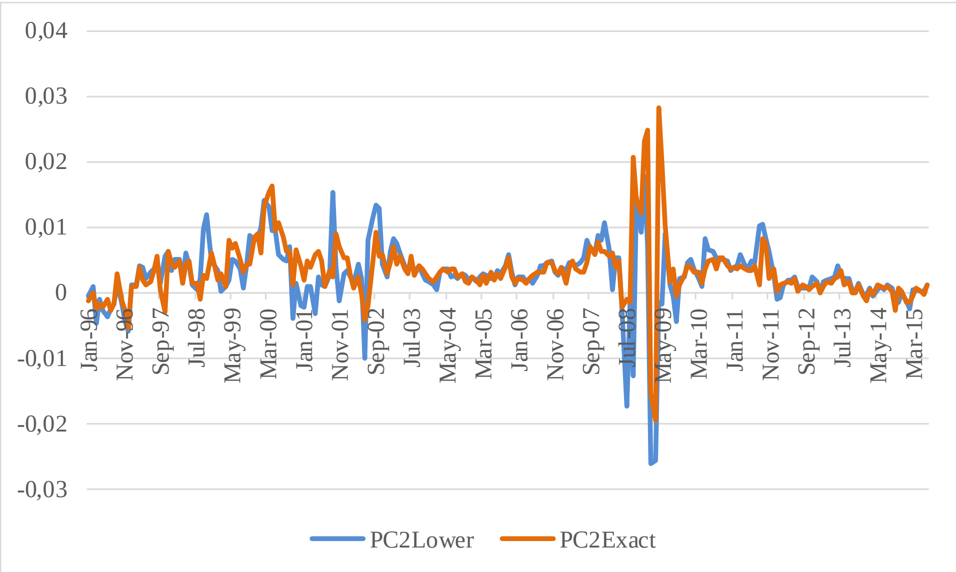


Figure 5.2 Lower Bound and Exact Second Principal Components. January 1996-2015.



Appendix. The Behavior and Factor Structure of Expected Returns Using the Model-Free Implied Variance of Jiang and Tian (2005)

Martin (2017) shows how to precisely relate the market's expected risk premium to its risk-neutral variance. The resulting volatility index is called SVIX. The square of this index measures risk-neutral variance. On other hand, the square of VIX measures risk-neutral entropy. Previous literature does not report a detailed analysis of the empirical differences generated by both procedures. This appendix covers partially this gap.

We next approximate risk-neutral variances following the insight of Britten-Jones and Neuberger (2000), who derived the model-free option implied variance under diffusion assumptions. Jiang and Tian (2005) extend their paper to show that their method is also valid in a jump-diffusion framework and, therefore, their methodology is a model-free procedure. We obtain the model-free implied variance denoted $MFIV_{t,t+\tau}^{j}$ by the following integral over a continuum of strikes:

$$MFIV_{jt,t+\tau} = 2R_{ft,t+\tau}\int_{0}^{\infty}\frac{C_{jt,t+\tau}\left(K\right)-\max\left(S_{jt}-K/R_{ft,t+\tau},0\right)}{K^{2}}dK \ , \qquad \text{(A.1)}$$

where $C_{jt,t+\tau}\left(K\right)$ is the price at time *t* of a *τ*-maturity call option on either an asset or an index *j* with strike *K*, and S_{jt} is the spot price of asset *j* at time *t* minus the present value of all expected future dividends to be paid before the option maturity. Specific implementation of equation (A.1) follows the approach of Jiang and Tian (2005), and the details are described in González-Urteaga and Rubio (2016).

The difference between both measures is that the risk-neutral variance derived by Martin (2017) is the price of a portfolio of out-the-money options equally-weighted by strike. Expression (A.1) is not the theoretical right measure to calculate risk-neutral variance. It represents the price of a portfolio of out-of-the-money options weighed by the inverse of the square of their strikes. To see this note that we can write expression (A.1) as

$$MFIV_{jt,t+\tau} = 2R_{ft,t+\tau} \left[\int_{0}^{F_{jt,t+\tau}} \frac{1}{K^2} P_{jt,t+\tau}(K)\, dK + \int_{F_{jt,t+\tau}}^{\infty} \frac{1}{K^2} C_{jt,t+\tau}(K)\, dK \right], \quad \text{(A.2)}$$

which resembles equation (7) but with a different weighting scheme.

From the estimation of equation (A.1) for each asset in the sample, we follow the same procedure of Section 4. We construct 20 risk-neutral variance-sorted portfolios. As before, P1 contains the assets with the lowest risk-neutral entropy, and P20 includes the stocks with the highest risk-neutral entropy. We update the components every month. We also obtain the corresponding realized returns of the 20 portfolios. Table A.1 contains the descriptive statistics. Note that, in all cases, the patterns of average, volatility, and betas are the same as in Table 1. However, the range of all descriptive statistics is higher than under Martin's procedure. P1 presents lower average statistics and P20 displays higher numbers in all cases. The average (lower bound) expected market excess return is 0.33% or 3.96% on annual basis.

In Table A.2, we report the factor structure obtained from (A.1). The results are very similar. The first two principal components explain practically the same percentage under both procedures. In this case, the first principal component captures a slightly lower variability than before. Table A.3 contains the temporal determinants of the first two principal components. In Panel A of Table A.3, we show that the same four factors, QMJ, BAB, DEF and the market VRP, plus the market explain 60% of the variability of the first principal component. Once again, the results are very similar under both methodologies. The main difference comes from the second principal component. If under the correct procedure, the MOM factor was practically the only factor significantly explaining the time-varying behaviour of the second principal component, we now find that market-wide illiquidity and the market VRP explain the second component. In any case, as before, we are able to capture only 6.6% of the variability of the second component. Finally, in Table A.4, we show that the cross-sectional variability of the approximate (lower bound) expected excess returns extracted from the model-free implied variance is explained by the same set of factors. The traditional *R*-square is 97.7% instead of 98.2% for the results obtained under expression (7). The difference comes from the corrected KRS *R*-square. Under the correct procedure, this is 83.6% but in Table A.4 is lower and equal to 60% approximately. The reason is that the pricing error of portfolio P20 is 0.47% under expression (7) but 0.61% under expression (A.1).

We can therefore conclude that for most empirical results regarding the behaviour of expected returns, both procedures generate similar results. Of course, the only valid theoretical approach if we want to estimate risk-neutral variance is provided by expression (7).

Table A.1. Descriptive Statistics of Approximate (Lower Bound) Expected Risk Premia and Realized Returns for 20 Portfolios Sorted by Model-Free Implied Variance (Jiang & Tian): January 1996-July 2015.

	(Lower Bound) Expected Risk Premium			Realized Returns		
	Average	Volatility	Market ERP Beta	Average	Volatility	Market Beta
P1	0.0024	0.0026	0.5892	0.0117	0.0262	0.3103
P2	0.0034	0.0034	0.7778	0.0131	0.0321	0.4150
P3	0.0040	0.0040	0.9195	0.0131	0.0339	0.4712
P4	0.0046	0.0046	1.0849	0.0134	0.0373	0.5925
P5	0.0051	0.0051	1.2310	0.0144	0.0386	0.6274
P6	0.0057	0.0056	1.3863	0.0135	0.0400	0.6803
P7	0.0062	0.0061	1.5310	0.0129	0.0446	0.8099
P8	0.0068	0.0065	1.6796	0.0145	0.0469	0.8063
P9	0.0073	0.0070	1.8114	0.0137	0.0490	0.9109
P10	0.0080	0.0075	1.9624	00133	0.0505	0.9251
P11	0.0086	0.0081	2.1398	0.0129	0.0548	0.9848
P12	0.0094	0.0090	2.3919	0.0090	0.0566	1.0410
P13	0.0103	0.0098	2.6306	0.0127	0.0569	0.9790
P14	0.0115	0.0112	2.9947	0.0112	0.0636	1.1718
P15	0.0130	0.0131	3.5025	0.0148	0.0650	1.2093
P16	0.0149	0.0156	4.1424	0.0107	0.0687	1.2992
P17	0.0176	0.0190	5.0712	0.0096	0.0755	1.3344
P18	0.0220	0.0238	6.3440	0.0107	0.0886	1.6184
P19	0.0295	0.0329	8.8641	0.0088	0.0958	1.6240
P20	0.0682	0.0868	20.752	0.0167	0.1608	2.6661
MARKET	0.0033	0.0029	1.0000	0.0066	0.0449	1.0000

This table presents the descriptive statistics of 20 portfolios sorted by model-free implied variance and the market, represented by the Standard & Poor 100 Index. The first two columns show the mean and the volatility of the approximate (lower bound) expected risk premia. The third column is the sensitivity of the expected risk premium of the 20 portfolios to the expected market risk premium. The last three columns contain the mean, the volatility and the market beta of realized returns.

Table A.2. The Factor Structure of Approximate (Lower Bound) Expected Risk Premia for 20 Portfolios Sorted by Model-Free Implied Variance (Jiang & Tian). Percentage Explained by the First Five Principal Components: January 1996-July 2015.

	Panel A: Principal Components		Panel B: Principal Components Connor & Korajczyk	
	(Lower Bound) Expected Risk Premia	Realized Excess Returns	(Lower Bound) Expected Risk Premia	Realized Excess Returns
Factor (PC) 1	94.89	72.41	96.64	72.42
Factor (PC) 2	99.40	82.58	99.62	82.87
Factor (PC) 3	99.90	86.71	99.94	87.09
Factor (PC) 4	99.95	88.89	99.97	89.25
Factor (PC) 5	99.97	90.50	99.98	90.79

The two columns of Panel A show the percentage of the variability of the 20 x 20 variance-covariance matrix of approximate (lower bound) expected and realized excess returns of our 20 portfolios, respectively, explained by the first five principal components. The two columns of Panel B report the same percentages of the variability but of the *T x T* covariance matrix, in this case, computed following the procedure of Connor and Korajczyk (1988).

Table A.3. Determinants of the Factor Structure of the Approximate (Lower Bound) Expected Risk Premia (Jiang & Tian). January 1996-July 2015.

Panel A: Determinants of the First Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.033	-0.256	0.097	-0.304	0.253	0.281							
(13.39)	(-4.18)	(1.19)	(-2.83)	(2.11)	(1.90)							0.154
[7.25]	[-2.18]	[1.40]	[-1.78]	[2.51]	[1.73]							
0.033	-0.295	0.123	-0.396	0.297	0.348	-0.131						
(13.80)	(-4.77)	(1.53)	(-3.58)	(2.49)	(2.35)	(-2.87)						0.180
[7.30]	[-2.35]	[1.84]	[-2.39]	[2.89]	[2.39]	[-1.81]						
0.031							0.401					
(12.93)							(5.26)					0.102
[7.51]							[2.20]					
0.034								-0.162				
(13.50)								(-2.72)				0.027
[6.52]								[-1.32]				
0.033									0.149			
(13.59)									(4.12)			0.064
[7.13]									[2.40]			
-0.037										2.855		
(-6.07)										(12.10)		0.383
[-3.02]										[4.84]		
0.041											0.033	
(16.43)											(7.63)	0.196
[6.25]											[2.76]	
-0.020	-0.041	0.046	0.069	-0.132	0.077	-0.006	0.400	-0.232	0.021	2.400	0.022	
(-3.59)	(-0.74)	(0.78)	(0.75)	(-0.92)	(0.71)	(-0.17)	(2.86)	(-4.53)	(0.84)	(11.41)	(6.20)	0.601
[-2.57]	[-0.79]	[1.19]	[0.55]	[-1.12]	[0.79]	[-0.21]	[3.68]	[-2.47]	[0.93]	[6.40]	[3.29]	
-0.019	-0.085						0.254	-0.220		2.423	0.023	
(-3.75)	(-1.68)						(3.53)	(-5.09)		(12.36)	(6.50)	0.602
[-2.78]	[-1.70]						[3.43]	[-2.76]		[7.25]	[3.10]	
-0.021							0.321	-0.209		2.461	0.025	
(-4.03)							(5.36)	(-4.88)		(12.60)	(7.51)	0.599
[-2.88]							[3.59]	[-2.68]		[7.08]	[3.21]	

Panel B: Determinants of the Second Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.004	-0.033	-0.006	-0.029	-0.011	0.040							
(7.18)	(-2.34)	(-0.35)	(-1.18)	(-0.42)	(1.18)							0.024
[4.59]	[-1.91]	[-0.38]	[-1.12]	[-0.47]	[1.35]							
0.004	-0.030	-0.008	-0.023	-0.015	0.035	0.009						
(7.07)	(-2.09)	(-0.44)	(-0.88)	(-0.53)	(1.03)	(0.86)						0.023
[4.41]	[-1.77]	[-0.46]	[-0.93]	[-0.57]	[1.37]	[0.63]						
0.004							0.025					
(6.94)							(1.48)					0.005
[4.43]							[1.08]					
0.004								-0.010				
(7.21)								(-0.79)				-0.002
[4.14]								[-0.49]				
0.004									0.021			
(7.35)									(2.69)			0.026
[4.78]									[3.65]			
0.002										0.057		
(1.48)										(0.90)		-0.001
[0.94]										[0.49]		
0.005											0.004	
(8.27)											(3.72)	0.052
[5.43]											[2.25]	
0.005	-0.021	-0.014	-0.022	0.021	0.039	0.014	-0.037	-0.021	0.012	0.010	0.003	
(2.52)	(-1.16)	(-0.73)	(-0.72)	(0.45)	(1.10)	(1.17)	(-0.80)	(-1.24)	(1.49)	(0.14)	(2.33)	0.054
[1.90]	[-0.81]	[-0.77]	[-0.65]	[0.44]	[1.13]	[1.08]	[-0.72]	[-0.99]	[2.07]	[0.10]	[1.18]	
0.005									0.016		0.003	
(8.19)									(2.10)		(3.30)	0.066
[5.45]									[2.90]		[1.94]	

This table shows the estimated coefficients of time-series regressions of each of the two first principal components on alternative state variables. The first six variables are the intercept and the Fama and French (2015) factors, MOM is the Momentum Factor of Carhart (1997), QMJ is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), BAB is the betting-against-beta factor of Frazzini and Pedersen (2014), P&S is the Pastor and Stambaugh (2003) illiquidity factor, DEF is the default premium, and VRP is the market variance risk premium. This is defined as the logarithm of the realized variance divided by the model-free implied variance. OLS *t*-statistics are reported in parenthesis and *t*-statistics based on HAC standard errors in brackets.

Table A.4. The Cross-Section of Portfolios sorted by Model-Free Implied Variance and a Multi-Factor Asset Pricing Model (Jiang & Tian). January 1996-July 2015.

Panel A: (Lower Bound) Expected Risk Premia

λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-square
0.0046	0.0080	-0.0179	-0.0087	-0.0019	-0.1150	0.977
(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	0.597
[0.000]	[0.011]	[0.000]	[0.070]	[0.021]	[0.217]	(0.00)

Panel B: Average Realized Excess Returns

λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-square
0.0121	-0.0041	0.0022	-0.0013	-0.0014	-0.0637	0.337
(0.000)	(0.371)	(0.546)	(0.830)	(0.356)	(0.507)	0.273
[0.000]	[0.409]	[0.637]	[0.875]	[0581]	[0.765]	(0.92)

This table reports the risk premia estimates from the two-pass cross-sectional regression of returns on rolling betas associated with the five aggregate risk factors: *m* represents the market excess return, *qmj* is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), *bab* denotes the betting-against-beta factor of Frazzini and Pedersen (2014), *def* is the default premium, and *vrp* is the market variance risk premium defined as the logarithm of the realized variance divided by the model-free implied variance. Panel A shows the results using (lower bound) expected risk premia, and Panel B employs excess realized returns. We report the *p*-value in parentheses and the Kan, Robotti, and Shanken (2013) adjusted *p*-value in brackets. The cross-sectional *R*-square reported in the first line is 1 minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. Kan, Robotti, and Shanken (2013) propose the cross-sectional *R*-square reported in the second line, and the corresponding (asymptotically valid) *p*-value is given in parentheses.